CP502 Advanced Fluid Mechanics
Problem Set #1 in Compressible Fluid Flow (June - Oct 2019)

Steady, quasi one-dimesional, isothermal, compressible flow of an ideal gas in a constant area duct with wall friction

$R_o = 8.31434$ kJ/(kmol K) $= 1.9858$ kcal/(kmol K)

(1) Starting from the mass and momentum balances, show that the differential equation describing the quasi one-dimensional, compressible, isothermal, steady flow of an ideal gas through a constant area pipe of diameter D and average Fanning friction factor $\bar{f}$ shall be written as follows:

$$\frac{4\bar{f}}{D}\,dx + \frac{2}{\rho\,u^2}\,dp + \frac{2}{u}\,du \;=\; 0 \tag{1.1}$$

where p, ρ and u are the respective pressure, density, and velocity at distance x from the entrance of the pipe.

(2) Show that the differential equation of Problem (1) can be converted into

$$\frac{4\bar{f}}{D}\,dx \;=\; -\left(\frac{2}{RT\,(\dot{m}/A)^2}\right)p\,dp + \frac{2}{p}\,dp \tag{1.2}$$

which in turn can be integrated to yield the following design equation:

$$\frac{4\bar{f}L}{D} \;=\; \frac{p^2}{RT\,(\dot{m}/A)^2}\left(1-\frac{p_L^2}{p^2}\right) + \ln\left(\frac{p_L^2}{p^2}\right), \tag{1.3}$$

where p is the pressure at the entrance of the pipe, p_L is the pressure at length L from the entrance of the pipe, R is the gas constant, T is the temperature of the gas, $\dot{m}$ is the mass flow rate of the gas flowing through the pipe, and A is the cross-sectional area of the pipe.

(3) Show that the design equation of Problem (2) is equivalent to

$$4\bar{f}\frac{L}{D} \;=\; \frac{1}{\gamma M^2}\left(1-\frac{M^2}{M_L^2}\right) + \ln\left(\frac{M^2}{M_L^2}\right), \tag{1.4}$$

where M is the Mach number at the entry and M_L is the Mach number at length L from the entry.

(4) Nitrogen (γ=1.4 and molecular mass=28) is to be fed through a 15 mm-id commercial steel pipe 11.5 m long to a synthetic ammonia plant. Calculate the downstream pressure in the line for a flow rate of 1.5 mol/s, an upstream pressure of 600 kPa, and a temperature of 27^{o}C throughout. The average Fanning friction factor may be taken as 0.0066. (*Answer:* $\approx$ *506 kPa*)

Rework the above in terms of Mach number and determine the downstream Mach number.

(5) Explain why the design equations of Problems (1), (2) and (3) are valid only for fully turbulent flow and not for laminar flow.

(6) Starting from the differential equation of Problem (2), or otherwise, prove that p, the pressure, in a quasi one-dimensional, compressible, isothermal, steady flow of an ideal gas in a constant area pipe with wall friction should always satisfies the following condition:

$$p \; > \; (\dot{m}/A)\,\sqrt{RT} \;\; \text{in flows where } p \text{ decreases along the flow direction,} \tag{1.5}$$

and

$$p \; > \; (\dot{m}/A)\,\sqrt{RT} \;\; \text{in flows where } p \text{ increases along the flow direction.} \tag{1.6}$$

(7) Air enters a horizontal constant-area pipe at 40 atm and 97^{o}C with a velocity of 500 m/s. What is the limiting pressure for isothermal flow? (*Answer: 61.4 atm*)

It can be observed that in the above case the pressure increases in the direction of flow. Is such flow physically realizable?

If yes, explain how the flow is driven along the pipe.

(8) Show that the equations in Problem 6 are equivalent to the following:

$$M \; < \; 1/\sqrt{\gamma} \quad \text{in flows where } p \text{ decreases along the flow direction} \tag{1.7}$$
$$M \; > \; 1/\sqrt{\gamma} \quad \text{in flows where } p \text{ increases along the flow direction} \tag{1.8}$$

(9) Show that when the flow has reached the limiting pressure $p^* = (\dot{m}/A)\,\sqrt{RT}$, or the limiting Mach number $M^* = 1/\sqrt{\gamma}$, the length of the pipe across which such conditions are reached, denoted by L_{max}, shall satisfy the following equation:

$$\frac{4\,\bar{f}\,L_{max}}{D} \; = \; \left(\frac{p^2}{p{*}^2} - 1\right) + \ln\left(\frac{p{*}^2}{p^2}\right) \; = \; \frac{1-\gamma M^2}{\gamma M^2} \; + \; \ln\left(\gamma M^2\right) \tag{1.9}$$

where pressure p and Mach number M are the conditions of the flow at the entrance of the pipe.

(10) Determine the isothermal mass flow rate of air in a pipe of 10-mm-i.d. and 1 m long with upstream condition of 1 MPa and 300 K with a exit pressure low enough to choke the flow in the pipe assuming an average Fanning friction factor of $\bar{f} = 0.0075$. (*Answer: 0.1116 kg/s*)

Determine also the exit pressure. (*Answer: 0.417 MPa*)

Given $\mu = 2.17\times10^{-5}$ kg/m s, calculate the Reynolds number of the flow to check if the given flow were turbulent.

(11) Air flows at a mass flow rate of 9.0 kg/s isothermally at 300 K through a straight rough duct of constant cross-sectional area 1.5×10^{-3} m^2. At one end A the pressure is 6.5 bar and at the other end B the pressure is 8.5 bar. Determine (i) the velocities u_A and u_B, (ii) the force acting on the duct wall, and (iii) the rate of heat transfer through the duct wall. (*Answers: (i) 794.8 & 607.8 m/s; (ii) 1383 N; (iii) 1180.3 kW*)

In which direction is the gas flowing? (*Answer: A to B*)

(12) Gas produced in a coal gasification plant (molecular weight = 0.013 kg/mol; $\mu = 10^{-5}$ kg/m s; $\gamma = 1.36$) is sent to neighbouring industrial users through a bare 15-cm-i.d. commercial steel pipe 100 m long. The pressure gauge at one end of the pipe reads 1 MPa absolute. At the other end it reads 500 kPa. The temperature is 87^{o}C. Estimate the flow rate of coal gas through the pipe? (*Answer: ≈ 9.4 kg/s*)

Additional Data: $\epsilon = 0.046$ mm for commercial steel. For fully developed turbulent flow in rough pipes, the average Fanning friction factor can be found by use of $(1/\sqrt{\bar{f}}) = 4\log_{10}(3.7\,d/\epsilon)$.

(13) Ethylene flows through a pipeline 10 km long to a receiving station A. At a point 3 km from A, a spur leads off the main pipeline and runs 5 km to a receiving station B. The internal diameter of the main pipeline is 0.20 m and that of the spur is 0.15 m. The flow rates into A and B are regulated by valves at these locations. If the pressure immediately upstream of valve A is 3.88 bar (absolute) and that at B is 3.69 bar when the flow rate into B is 0.63 kg/s, calculate the pressure at the beginning of the main pipeline, assuming that flow in the pipeline is isothermal at a temperature of 20^{o}C. (*Answers: 6.71 bar; pressure at junction is 4.5 bar*)

Additional Data: Specific volume of ethylene at 20^{o}C, 1 bar = 0.870 m^3/kg and Fanning friction factor = 0.0045.

(14) Methane (molecular mass = 16 kg/kmol) is supplied to a gas pipeline of diameter 0.50 m and length 40 km at a mass flow rate of 13.0 kg/s and a pressure of 11.0 bar. Because of the large heat-transfer area available it may be assumed that the temperature of the gas remains constant at the ambient value of 27^{o}C. Assuming that f = 0.005, evaluate (i) the pressure at exit, (ii) the velocities at inlet and exit, and (iii) the rate of heat transfer to the gas.

(*Answers: (i) 3.42 bar; (ii) 9.38 & 30.18 m/s; (iii) 411.5 J/kg*)

Also, evaluate (i) the entropy change resulting from the heat transfer and (ii) the total entropy change. Comment on the relative magnitude of these last two quantities. (*Answers: (i) 1.371 J/kg K; (ii) 607 J/kg K*)

Additional Data: It may be assumed without proof that $ds = c_P(dT/T) - R(dp/p)$.

References:

1. Anderson, J.D. Jr., *Modern Compressible Flow*, 2nd Edition.
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3. Hughes, W.F. & Brighton, J.A., *Fluid Dynamics*, 2nd Edition.
4. Levenspiel, O., *Engineering Flow and Heat Exchange.*
5. Shapiro, A.H., *The Dynamics and Thermodynamics of Compressible Fluid Flow - Volume 1.*
6. Yahya, S.M., *Fundamentals of Compressible Fluid Flow - SI Units.*