Isentropic Flow of an Ideal Gas

Considered here is the one-dimensional, isentropic, compressible, steady flow of an ideal gas with constant specific heat and with isentropic exponent γ.

- Relations between temperature T, pressure p and density ρ:

$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma}$$

- T, p and ρ as functions of Mach number M and of stagnation values T_0, p_0 and ρ_0:

$$\begin{aligned}\frac{T_0}{T} &= 1 + \frac{\gamma-1}{2}M^2\\ \frac{p_0}{p} &= \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}\\ \frac{\rho_0}{\rho} &= \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}\end{aligned}$$

- Mass Flow Rate (dm/dt) in terms of area A of the flow cross-section and other properties:

$$\begin{aligned}\frac{dm}{dt} &= A\,M\,p\sqrt{\frac{\gamma}{RT}}\\ &= A\,M\,p_0\sqrt{\frac{\gamma}{RT_0}}\left(\frac{1}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma+1}{2\,(\gamma-1)}}\end{aligned}$$

- Area Ratio, for flow through a variable area pipe, as a function of M with reference area as A^* at sonic conditions (i.e. at $M = 1$):

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2\,(\gamma-1)}}$$

- Table for Isentropic Flow of an Ideal Gas with γ = 1.4:

M	T/T_0	p/p_0	A/A^*
0.00	1.000	1.000	∞
0.05	0.999	0.998	11.592
0.10	0.998 00	0.993 03	5.8218
0.15	0.996	0.984	3.910
0.20	0.992 06	0.972 50	2.9635
0.25	0.987	0.957	2.403
0.30	0.982 32	0.939 47	2.0351
0.35	0.976	0.918	1.778
0.40	0.968 99	0.895 62	1.5901
0.45	0.961	0.870	1.448
0.50	0.952 38	0.843 02	1.3398
0.55	0.943	0.814	1.255
0.60	0.932 84	0.784 00	1.1882
0.65	0.922	0.753	1.135
0.70	0.910 75	0.720 92	1.094 37
0.75	0.898	0.688	1.062
0.80	0.886 52	0.656 02	1.038 23
0.85	0.874	0.623	1.020
0.90	0.860 58	0.591 26	1.008 86
0.95	0.847	0.559	1.002
1.00	0.833 33	0.528 28	1.000 00
1.10	0.805 15	0.468 35	1.007 93
1.20	0.776 40	0.412 38	1.030 44
1.30	0.747 38	0.360 92	1.066 31
1.40	0.718 39	0.314 24	1.1149
1.50	0.689 65	0.272 40	1.1762
1.60	0.661 38	0.235 27	1.2502
1.70	0.633 72	0.202 59	1.3376
1.80	0.606 80	0.174 04	1.4390
1.90	0.580 72	0.149 24	1.5552
2.00	0.555 56	0.127 80	1.6875
2.10	0.531 35	0.109 35	1.8369
2.20	0.508 13	0.093 52	2.0050
2.30	0.485 91	0.079 97	2.1931
2.40	0.464 68	0.068 40	2.4031
2.50	0.444 44	0.058 53	2.6367
2.60	0.425 17	0.050 12	2.8960
2.70	0.406 84	0.042 95	3.1830
2.80	0.389 41	0.036 85	3.5001
2.90	0.372 86	0.031 65	3.8498
3.00	0.357 14	0.027 22	4.2346
3.50	0.289 86	0.013 11	6.7896
4.00	0.238 10	0.006 58	10.719
4.50	0.198 02	0.003 46	16.562
5.00	0.166 67	0.001 89	25.000

Flow of an Ideal Gas with Normal Shock

Considered here is the one-dimensional, adiabatic, steady flow of an ideal gas (with constant specific heat and with isentropic exponent γ) with a thin normal shock wave.

- Fluid properties at state y (i.e. just downstream of the shock) are related to fluid properties at state x (i.e. just upstream of the shock) by the following relations:

$$\frac{T_y}{T_x} = \frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2} = \left(\frac{1+\frac{\gamma-1}{2}M_x^2}{\frac{\gamma+1}{2}M_x^2}\right)\left(\frac{2\gamma M_x^2 - (\gamma-1)}{\gamma+1}\right)$$

$$\frac{p_y}{p_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} = \frac{2\gamma M_x^2 - (\gamma-1)}{\gamma+1}$$

$$\frac{\rho_y}{\rho_x} = \frac{T_x}{T_y}\frac{p_y}{p_x} = \frac{\frac{\gamma+1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_x^2}$$

$$\frac{T_{0y}}{T_{0x}} = 1$$

$$\frac{p_{0y}}{p_{0x}} = \left(\frac{\frac{\gamma+1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_x^2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{2\gamma M_x^2 - (\gamma-1)}{\gamma+1}\right)^{\frac{-1}{\gamma-1}}$$

- Mach number just downstream of the shock M_y as a function of Mach number just upstream of the shock M_x:

$$M_y^2 = \frac{1+\frac{\gamma-1}{2}M_x^2}{\gamma M_x^2 - \frac{\gamma-1}{2}}$$

- Entropy change across the shock:

$$s_y - s_x = c_p \ln\frac{T_y}{T_x} - R\ln\frac{p_y}{p_x} = -R\ln\frac{p_{0y}}{p_{0x}}$$

- Table for Normal Shocks in an Ideal Gas with γ = 1.4:

M_x	M_y	p_y/p_x	T_y/T_x	p_{0y}/p_{0x}
1.00	1.000	1.000	1.000	1.000
1.05	0.953	1.119	1.033	0.999
1.10	0.912	1.245	1.065	0.999
1.12	0.896	1.297	1.077	0.998
1.14	0.882	1.349	1.090	0.997
1.16	0.868	1.403	1.103	0.996
1.18	0.855	1.458	1.115	0.995
1.20	0.842	1.513	1.128	0.993
1.22	0.829	1.569	1.140	0.991
1.24	0.818	1.627	1.153	0.988
1.26	0.807	1.686	1.166	0.985
1.28	0.796	1.745	1.178	0.983
1.30	0.786	1.805	1.191	0.979
1.32	0.776	1.866	1.203	0.976
1.34	0.766	1.928	1.216	0.972
1.36	0.757	1.991	1.229	0.967
1.38	0.748	2.055	1.242	0.963
1.40	0.739	2.120	1.255	0.958
1.42	0.731	2.186	1.267	0.953
1.44	0.723	2.253	1.281	0.947
1.46	0.716	2.320	1.294	0.942
1.48	0.708	2.389	1.307	0.936
1.50	0.701	2.458	1.320	0.929
1.52	0.694	2.529	1.333	0.923
1.54	0.687	2.600	1.347	0.916
1.56	0.681	2.672	1.361	0.907
1.58	0.675	2.746	1.374	0.903
1.60	0.668	2.820	1.388	0.895
1.70	0.641	3.205	1.458	0.856
1.75	0.628	3.406	1.495	0.834
1.80	0.616	3.613	1.532	0.813
1.85	0.605	3.826	1.569	0.790
1.90	0.595	4.045	1.608	0.767
2.00	0.577	4.500	1.687	0.721
2.50	0.513	7.125	2.137	0.499
3.00	0.475	10.333	2.679	0.328
3.50	0.451	14.125	3.315	0.213
4.00	0.435	18.500	4.047	0.138
4.50	0.423	23.548	4.875	0.0917
5.00	0.415	29.000	5.800	0.0617

Isothermal Flow of an Ideal Gas in Constant Area Ducts with Friction

Considered here is the one-dimensional, isothermal, compressible, steady flow of an ideal gas (with constant specific heat and with isentropic exponent γ) in a constant area duct with friction.

- The duct length L between the position where pressure is p and Mach number is M and the position where pressure is p_L and Mach number is M_L:

$$\frac{4\,\bar{f}\,L}{D} = \frac{p^2}{RT\,(\dot{m}/A)^2}\left(1-\frac{p_L^2}{p^2}\right)+\ln\left(\frac{p_L^2}{p^2}\right) = \frac{1}{\gamma M^2}\left(1-\frac{M^2}{M_L^2}\right)+\ln\left(\frac{M^2}{M_L^2}\right),$$

  where $\bar{f}$ is the average fanning friction factor over the length and D is the hydraulic diameter. The gas constant is R and temperature of the fluid is T. Mass flow rate is $\dot{m}$ and area of cross-section is A.

- The critical duct length L_{max} between the position where pressure is p and Mach number is M and the position where the limiting Mach number of $(1/\sqrt{\gamma})$ is reached at the critical pressure $p*$:

$$\frac{4\,\bar{f}\,L_{max}}{D} = \left(\frac{p^2}{p*^2}-1\right)+\ln\left(\frac{p*^2}{p^2}\right) = \frac{1-\gamma M^2}{\gamma M^2} + \ln\left(\gamma M^2\right).$$

- The invariables of the flow:

$$T = \text{constant}$$
$$\rho u = \frac{\dot{m}}{A}$$
$$\rho M = \frac{\dot{m}}{A}\sqrt{\frac{1}{\gamma RT}}$$
$$pM = \frac{\dot{m}}{A}\sqrt{\frac{RT}{\gamma}}$$

- Entropy difference between entropy s at M and the critical entropy $s*$ at $M = 1/\sqrt{\gamma}$:

$$s - s* = -R\,\ln\left(\frac{p}{p*}\right) = R\,\ln\left(\sqrt{\gamma}M\right).$$

- Table for Isothermal Flow of an Ideal Gas with γ = 1.4 in a Constant Area Duct with Friction:

M	$4\bar{f}L_{max}/D$
0.00	∞
0.05	279.06
0.10	66.160
0.12	44.699
0.14	31.847
0.16	23.573
0.18	17.953
0.20	13.975
0.25	7.993
0.30	4.865
0.35	3.068
0.40	1.968
0.45	1.267
0.50	0.807
0.52	0.670
0.54	0.554
0.56	0.455
0.58	0.370
0.60	0.299
0.65	0.1655
0.70	0.0808
0.75	0.0309
0.80	0.0063
0.81	0.0037
0.82	0.0019
0.83	0.0007
0.84	0.0001
0.8452	0.0000
0.85	0.0001
0.90	0.0076
0.95	0.0253
1.00	0.0508
1.20	0.197
1.50	0.465
1.60	0.556
1.70	0.645
1.80	0.733
1.90	0.818
2.00	0.901
2.50	1.283
3.00	1.613
3.50	1.900
4.00	2.154
4.50	2.380
5.00	2.584