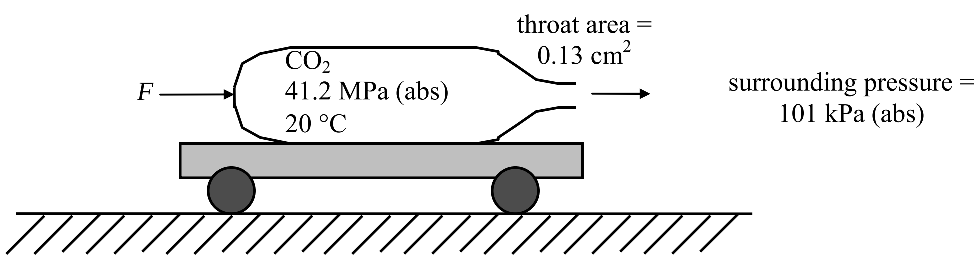
Practice Problems on Compressible Flow

comp_02

A CO_2 cartridge is used to propel a small rocket cart. Compressed CO_2, stored at a pressure of 41.2 MPa (abs) and a temperature of 20 °C, is expanded through a smoothly contoured converging nozzle with a throat area of 0.13 cm^2. Assume that the cartridge is well insulated and that the pressure surrounding the cartridge is 101 kPa (abs). For the given conditions,

a. Calculate the pressure at the nozzle throat.
b. Evaluate the mass flow rate of carbon dioxide through the nozzle.
c. Determine the force, *F*, required to hold the cart stationary.
d. Sketch the process on a *T*-*s* diagram.
e. For what range of cartridge pressures will the flow through the nozzle be choked?
f. Will the mass flow rate from the cartridge remain constant for the range of cartridge pressures you found in part (e)? Explain your answer.
g. Write down (but do not solve) the differential equations describing how the pressure within the tank varies with time while the flow is choked.

Note: For CO_2, the ideal gas constant is 189 J/(kg-K) and the specific heat ratio is 1.30.



Answer(s):

$\therefore p_E = 22.5 \text{ MPa}$

$\therefore \dot{m} = 1.52 \text{ kg/s}$

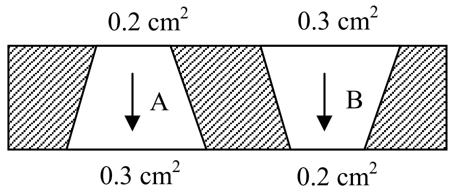
$\therefore F = 671 \text{ N}$

$\therefore p_0 \geq 185 \text{ kPa}$

comp_15

The orientation of a hole can make a difference. Consider holes A and B in the figure below which are identical but reversed. For the given air properties on either side, compute the mass flow rate through each hole and explain why they are different.

p_0 = 150 kPa (abs)
T_0 = 20 °C



p_{back} = 100 kPa (abs)

Answer(s):

$\therefore$	$\dot{m}$	=	6.78e-3 kg/s
$\therefore$	$\dot{m}$	=	7.08e-3 kg/s

comp_16

A large tank supplies helium through a converging-diverging nozzle to the atmosphere. Pressure in the tank remains constant at 8.00 MPa (abs) and temperature remains constant at 1000 K. There are no shock waves in the nozzle. The nozzle is designed to discharge at an exit Mach number of 3.5. The exit area of the nozzle is 100 mm^2. Note that for helium the specific heat ratio is 1.66 and the ideal gas constant is 2077 J/(kg·K).

a. Determine the pressure at the exit of the converging/diverging nozzle.
b. Determine the mass flow rate through the device.
c. Sketch the flow process from the tank through the converging/diverging nozzle to the exit on a *T-s* diagram.

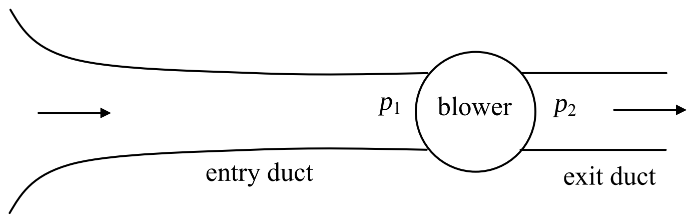
Answer(s):

p_e = 137 kPa

$\dot{m}$ = 9.59e-6 kg/s

comp_23

An air blower takes air from the atmosphere (100 kPa and 293 K) and ingests it through a smooth entry duct so that the losses are negligible. The cross-sectional area of the entry duct just upstream of the blower and that of the exit duct are both 0.01 m^2.



The pressure ratio, p_2/p_1, across the blower is 1.05 and the exit pressure is equal to atmospheric pressure. The air is assumed to behave isentropically upstream of the blower. Find:

a. the velocity of the air entering the blower, and
b. the mass flow rate of air through the system.

Answer(s):

V_1	=	90 m/s
$\dot{m}$	=	1.03 kg/s

comp_24

A pitot tube is used to measure the velocity of air. At low speeds, we can reasonably treat the air as an incompressible fluid; however, at high speeds this assumption is not very good due to compressibility effects. At what Mach number does the incompressibility assumption become inaccurate for engineering calculations? Justify your answer with appropriate calculations.

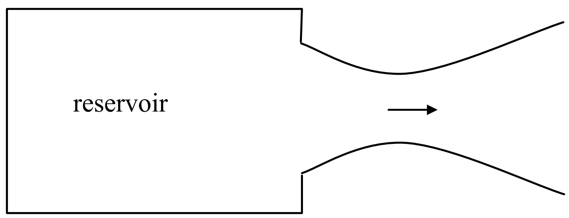
Answer(s):

$$\varepsilon \equiv \frac{\left(V_\infty\right)_{\substack{\text{isentropic}\\ \text{ideal gas}}} - \left(V_\infty\right)_{\text{incompressible}}}{\left(V_\infty\right)_{\substack{\text{isentropic}\\ \text{ideal gas}}}} = 1 - \sqrt{\frac{2}{\gamma \text{Ma}_\infty^2}\left[\left(1+\frac{\gamma-1}{2}\text{Ma}_\infty^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]}$$

if we consider <1% error acceptable, the incompressibility assumption is valid for Ma_∞ <≈ 0.3.

comp_25

A rocket engine can be modeled as a reservoir of gas at high temperature feeding gas to a convergent/divergent nozzle as shown in the figure below.



For the questions below, assume the following:

1. The temperature in the reservoir is 3000 K.
2. The exhaust gases have the same properties as air: γ=1.4, R=287 J/(kg·K).
3. The exit Mach number is 2.5.
4. The rocket operates at design conditions (no shock waves or expansion waves present) where the surrounding pressure is $1*10^5$ Pa.
5. The area of the exit is $1*10^{-4}$ m^2.

Determine:

a. the temperature of the flow at the exit,
b. the pressure in the reservoir,
c. the throat area,
d. the mass flow rate out of the rocket,
e. the thrust produced by the rocket, and
f. sketch the process on a *T*-*s* diagram.

Answer(s):

T_E = 1333 K

$p_0 = 1.709*10^6$ Pa

$A_T = 3.79*10^{-5}$ m^2

$\dot{m} = 4.776*10^{-2}$ kg/s

F = 87.4 N

comp_32

A small, solid fuel rocket motor is tested on a horizontal thrust stand at atmospheric conditions. The chamber (essentially a large tank) absolute pressure and temperature are maintained at 4.2 MPa (abs) and 3333 K, respectively. The rocket's converging-diverging nozzle is designed to expand the exhaust gas isentropically to an absolute pressure of 69 kPa. The nozzle exit area is 0.056 m^2. The gas may be treated as a perfect gas with a specific heat ratio of 1.2 and an ideal gas constant of 300 J/(kg·K). Determine, for design conditions:

a. the mass flow rate of propellant gas, and
b. the thrust force exerted on the test stand.

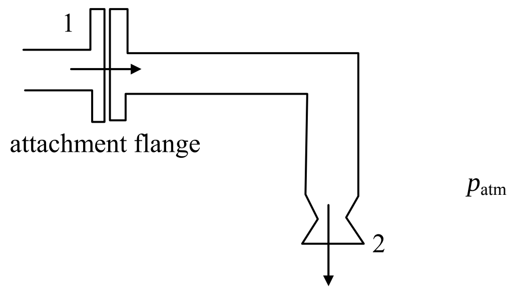
Answer(s):

$\therefore \dot{m} = 18.69 \text{ kg/s}$

$\therefore T = 4.380\text{e}4 \text{ N}$

comp_33

A steady flow of air passes through the elbow-nozzle assembly shown. At the inlet (1), the pipe diameter is D_1 = 0.1524 m and the air properties are p_1 = 871.7 kPa (abs), T_1 = 529.0 K, and V_1 = 230.4 m/s. The air is expanded through a converging-diverging nozzle discharging into the atmosphere where p_{atm} = 101.3 kPa (abs). At the nozzle exit (2), the nozzle diameter is D_2 = 0.2280 m and the air properties are T_2 = 475.8 K and V_2 = 400.0 m/s.



a. Is the flow through the elbow-nozzle assembly adiabatic?
b. Determine the components of the force in the attachment flange required to hold the elbow-nozzle assembly in place. You may neglect the effects of gravity.

Answer(s):

the flow can be considered adiabatic in going from 1 to 2

F_x = -19.60 kN

F_y = -9.647 kN

comp_36

A rocket engine is designed to operate at a pressure ratio (inlet reservoir pressure/back pressure) of 37. Find:

a. the ratio of the exit area to the throat area which is necessary for the supersonic exhaust to be correctly expanded,
b. the Mach number of the exit flow under correctly expanded conditions,
c. the lowest pressure ratio (p_0/p_b) at which the same nozzle would be choked, and
d. the pressure ratio (p_0/p_b) at which there would be a normal shock wave at the exit.

Assume the specific heat ratio of the gas is 1.4.

Answer(s):

$\text{Ma}_e = 3.0$

$A_e/A_t = 4.3$

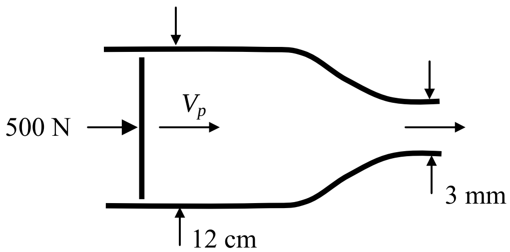
$p_0/p_b = 1.01$

$p_{01}/p_b = 3.6$

comp_38

A force of 500 N pushes a piston of diameter 12 cm through an insulated cylinder containing air at 20 °C. The exit diameter is 3 mm and the atmospheric pressure is 1 atm. Estimate:

1. the exit velocity,
2. the velocity near the piston (V_p), and
3. mass flow rate out of the device.



Answer(s):

$\therefore V_2 = 241 \text{ m/s}$

$\therefore V_1 = 0.116 \text{ m/s}$

$\therefore \dot{m} = 2.27 * 10^{-3} \text{ kg/s}$

comp_40

Air flows steadily between two sections in a long straight portion of 10.2 cm diameter pipe. The temperature and pressure at the inlet are 27 °C and 590 kPa (gage), and at the outlet are 10 °C and 26 kPa (gage). Calculate:

a. the change in specific internal energy between the inlet and outlet,
b. the change in the specific enthalpy between the inlet and outlet,
c. the change in density between the inlet and outlet, and
d. the change in specific entropy between the inlet and outlet.
e. Would you expect compressibility effects to be important for this flow?

State any major assumptions you make.

Answer(s):

$u_2 - u_1 = -12.2 \text{ kJ/kg}$

$h_2 - h_1 = -17.1 \text{ kJ/kg}$

$\therefore \rho_2 - \rho_1 = -6.46 \text{ kg/m}^3$

$s_2 - s_1 = 428 \text{ J/kg}$

Since the change in density is large compared to the flow's initial density,

$$\frac{\rho_2 - \rho_1}{\rho_1} = \frac{-6.46 \text{ kg/m}^3}{8.03 \text{ kg/m}^3} = -0.805 \quad (1)$$

we should expect that compressibility effects are significant.

comp_41

Air enters a turbine in steady flow at 0.5 kg/s with negligible velocity. Inlet conditions are 1300 °C and 2.0 MPa (abs). The air is expanded through the turbine to atmospheric pressure. If the actual temperature and velocity at the turbine exit are 500 °C and 200 m/s, determine the power produced by the turbine. Determine the change in specific entropy for the process. Label state points on a *Ts* diagram for this process.

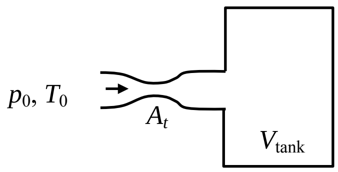
Answer(s):

$$\boxed{\dot{W}_{\substack{\text{extracted}\\\text{from air}}} = 390 \text{ kW}}$$

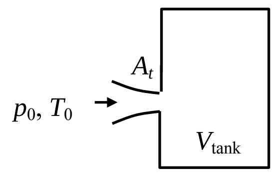
$$\boxed{\therefore \Delta s = 140 \ \text{J}/(\text{kg}\cdot\text{K}) > 0}$$

comp_44

Which nozzle will fill the tank faster (or will they fill at the same rate), assuming that the tank is initially evacuated? Justify your answer. The upstream stagnation properties, throat areas, and tank volumes are identical in both cases.



converging-diverging nozzle



converging nozzle

Answer(s):

The converging-diverging nozzle will fill the tank faster.

isentropic1D_02

Air flows isentropically through a converging nozzle. At a section where the nozzle area is 0.013 ft^2, the local pressure, temperature, and Mach number are 60 psia, 40 °F, and 0.52, respectively. The back pressure is 30 psia. Determine:

a. the Mach number at the throat,
b. the mass flow rate, and
c. the throat area.

Answer(s):

$\boxed{Ma_T = 1}$

$\boxed{A_T = A^* = 9.97 \times 10^{-3} \text{ ft}^2}$

$\boxed{\dot{m}_{\text{choked}} = 2.40 \text{ lb}_m/\text{s}}$

isentropic1D_04

Air flows isentropically in a converging-diverging nozzle, with exit area of 0.001 m^2. The nozzle is fed from a large plenum where the stagnation conditions are 350 K and 1.0 MPa (abs). The nozzle has a design back pressure of 87.5 kPa (abs) but is operating at a back pressure of 50.0 kPa (abs). Assuming the flow within the nozzle is isentropic, determine:

a. the exit Mach number, and
b. the mass flow rate through the nozzle.

Answer(s):

$\boxed{Ma_E = 2.24}$.

$\boxed{\dot{m} = 1.04\ \text{kg/s}}$

isentropic1D_06

The control systems for some smaller space vehicles use nitrogen (specific heat ratio of 1.4 and gas constant of 296.8 J/(kg·K)) from a high-pressure bottle. When the vehicle has to be maneuvered, a valve is opened allowing nitrogen to flow out through a nozzle thus generating thrust. In a typical system, the reservoir pressure and temperature are about 1.6 MPa (abs) and 30 °C, respectively, while the pressure at the nozzle exit plane is about 10.5 kPa (abs). Assuming that the flow through the nozzle is isentropic:

a. determine the temperature and the velocity of the nitrogen in the nozzle exit plane.
b. If the thrust required to maneuver the vehicle is 1 kN, determine the area of the nozzle exit plane and the required mass flow rate of nitrogen.

Answer(s):

T_E	=	72 K
V_E	=	690 m/s
A_E	=	4.1×10^{-3} m^2
$\dot{m}$	=	1.4 kg/s

isentropic1D_07

In wind-tunnel testing near Ma = 1, a small area decrease caused by model blockage can be important. Suppose the test section area is 1 m^2, with unblocked test conditions Ma = 1.10 and T = 20 °C.

a. What model area will first cause the test section to choke?

b. If the model cross section is 0.004 m^2 (0.4 % blockage), what percentage change in test section velocity results?

Answer(s):

0.008 m^2

% change = -2.2%.

isentropic1D_11

Oxygen (not air) enters a device with a cross-sectional area of 1 ft^2 (refer to this location as section 1) with a stagnation temperature of 1000 °R, stagnation pressure of 100 psia, and Mach number of 0.2. There is no heat transfer, work transfer, or losses as the gas passes through the device and expands to a pressure of 14.7 psia (section 2).

a. Determine the density, velocity, and mass flow rate at section 1.
b. Determine the Mach number, temperature, velocity, density, and area at section 2.
c. What force does the fluid exert on the device?

Answer(s):

$\rho_1 = 0.292\ lb_m/ft^3$

$V_1 = 293.3$ ft/s

$\dot{m} = 85.6\ lb_m/s$

$Ma_2 = 1.91$

$T_2 = 581.2$ °R

$V_2 = 2144$ ft/s

$\rho_2 = 0.0754\ lb_m/ft^3$

$A_2 = 0.53\ ft^2$

force the fluid exerts on the device is 7950 lb_f acting in the $+x$-direction

isentropic1D_12

Air, at a stagnation pressure of 7.20 MPa (abs) and a stagnation temperature of 1100 K, flows isentropically through a converging-diverging nozzle having a throat area of 0.01 m^2. Determine the speed and the mass flow rate at the downstream section where the Mach number is 4.0.

Answer(s):

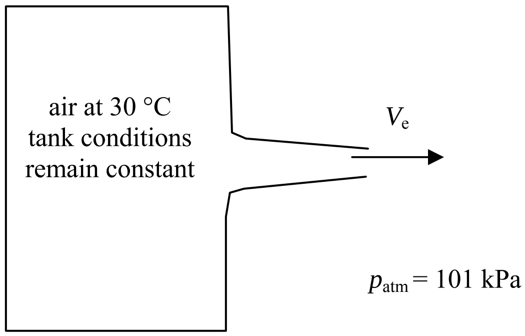
$V = 1298$ m/s

$\dot{m} = 87.6 \text{ kg/s}$

isentropic1D_13

The large compressed-air tank shown in the figure exhausts from a nozzle at an exit velocity of V_e=235 m/s. Assuming isentropic flow, compute:

a. the pressure in the tank
b. the exit Mach number



Answer(s):

Ma_e	=	0.71
p_0	=	141 kPa

isentropic1D_14

Air flows isentropically through a converging nozzle. At a section where the nozzle area is 0.013 ft^2, the local pressure, temperature, and Mach number are 60 psia, 40 °F, and 0.52, respectively. The back pressure is 30 psia. The Mach number at the exit, the mass flow rate, and the exit area are to be determined.

Answer(s):

$Ma_e = 1$

$A_e = 9.97*10^{-3}$ ft^2

$\dot{m} = 7.46*10^{-2}$ slug/s

isentropic1D_15

The Concorde aircraft flies at Ma≈2.3 at 11 km standard altitude. Estimate the temperature in °C at the front stagnation point. At what Mach number would it have a front stagnation point temperature of 450 °C?

Answer(s):

T_0 = 447 K = 174 °C

$\mathrm{Ma}_\infty = 3.4$

isentropic1D_16

A steady flow of air passes through a converging nozzle. At the nozzle inlet, the static pressure and temperature are p_1 = 150 kPa (abs), T_1 = 500 K, and V_1 = 150 m/s. At the nozzle exit, p_2 = 98.32 kPa (abs), T_2 = 453.2 K, and V_2 = 341.4 m/s. Assume steady, uniform flow, and that the air behaves as a perfect gas with γ = 1.4, R = 287 J/(kg·K), and c_p = 1005 J/(kg·K).

a. Is the flow through the nozzle adiabatic?
b. Is the flow through the nozzle isentropic?
c. Is the flow through the nozzle frictionless?

Support all of your answers.

Answer(s):

the flow must be adiabatic

the flow is not isentropic

the flow in this nozzle is not frictionless

isentropic1D_17

A certain fixed amount of gaseous fuel is to be fed steadily from a tank to the atmosphere through a converging nozzle. A young engineer comes to you with the following scheme: "Pressurize the tank to a pressure considerably higher than atmospheric pressure. At the fuel nozzle outlet the Mach number will then be equal to one. As long as the Mach number is one at the nozzle outlet, we will have the same mass flow rate." Do you agree with the young engineer, or do you send him back to the drawing board? Explain your answer.

Answer(s):

Back to the drawing board for the young engineer!

isentropic1D_18

Natural gas, with the thermodynamic properties of methane, flows in an insulated, underground pipeline of 0.6 m diameter. The gage pressure at the inlet to a compressor station is 0.5 MPa; the outlet pressure is 8.0 MPa (gage). The gas temperature and speed at inlet are 13 °C and 32 m/s, respectively. The compressor efficiency is 85%. Calculate the mass flow rate of natural gas through the pipeline. Evaluate the gas temperature and speed at the compressor outlet and the power required to drive the compressor.

Answer(s):

$\therefore \dot{m} = 3.7 * 10^{1}\ \text{kg/s}$

$T_o = 573$ K

$\therefore V_o = 4.7$ m/s

$\dot{W}_{\text{on gas}} = 23$ MW

isentropic1D_20

There is a long underground hallway between the Purdue Union and the Krannert School of Management (near the bowling alley). My kids and I call this hallway the "echo hallway" for obvious reasons. Using this echo, estimate the length of the hallway. [You actually need to perform this experiment to get the answer.]

Answer(s):

L_{echo} = 110±3 m

isentropic1D_21

A supersonic wind tunnel test section is designed to have a Mach number of 2.5 at a temperature of 60 °F and 5 psia. The fluid is air.

a. Determine the required inlet stagnation temperature and pressure.

b. Calculate the required mass flow rate for a test section area of 2.0 ft^2.

Answer(s):

$\boxed{p_0 = 85.4 \text{ psia}}$ and $\boxed{T_0 = 1170\ °\text{R}}$

$\boxed{\dot{m}_{TS} = 145\ \text{lb}_\text{m}/\text{s}}$

isentropic1D_22

A large tank contains 0.7 MPa, 27 °C air. The tank feeds a converging-diverging nozzle with a throat area of $6.45*10^{-4}$ m^2. At a particular point in the nozzle, the Mach number is 2.

a. What is the area at that point?
b. What is the mass flow rate at that point?

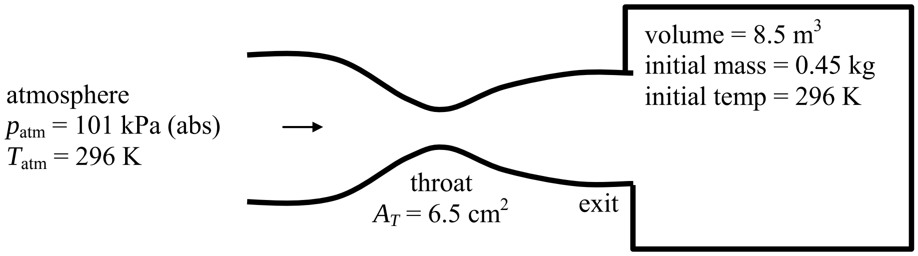
Answer(s):

$A = 1.09*10^{-3}$ m^2

$\dot{m} = 1.05$ kg/s

isentropic1D_23

An 8.5 m^3 vacuum tank is to be used to create a flow at an exit Mach number of $Ma_E = 2.0$ (refer to the figure below). A plug is put into the nozzle and the tank is evacuated until it contains 0.45 kg of air at a temperature of 296 K. When the plug is removed, air flows from the atmosphere into the tank through the converging-diverging nozzle. The throat area is $A_T = 6.5\ \text{cm}^2$.



a. Determine the design exit area.
b. Determine the initial pressure in the tank.
c. Determine the initial mass flow rate through the nozzle.
d. Determine the exit pressure, p_E, immediately after the flow begins.
e. Determine the tank pressure at which a normal shock wave will stand in the nozzle exit plane.

Answer(s):

$A_{E,d} = 11.0\ \text{cm}^2$

$p_{tank}(t = 0) = 4.50$ kPa (abs)

$\dot{m}_{choked} = 0.154$ kg/s

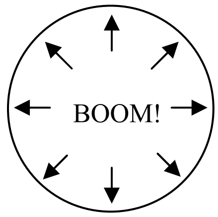
$p_{E,d} = 12.9$ kPa (abs).

58.1 kPa (abs).

normalshock_01

An explosion creates a spherical shock wave propagating radially into still air at standard conditions. A recording instrument registers a maximum pressure of 200 psig as the shock wave passes by. Estimate:

a. the speed of the shock wave with respect to a fixed observer in ft/sec
b. the wind speed following the shock with respect to a fixed observer in ft/sec



Answer(s):
4020 ft/s
3080 ft/s

normalshock_02

Stagnation pressure and temperature probes are located on the nose of a supersonic aircraft at 35,000 ft altitude. A normal shock stands in front of the probes. The temperature probe indicates $T_0 = 420$ °F behind the shock.

a. Calculate the Mach number and airspeed of the plane.
b. Find the static and stagnation pressures behind the shock.
c. Show the process and the static and stagnation points on a *T*-*s* diagram.

Answer(s):

$\mathrm{Ma}_1 = 2.48$

$V_1 = 2410$ ft/s

$p_2 = 24.2$ psia

$p_{02} = 29.1$ psia

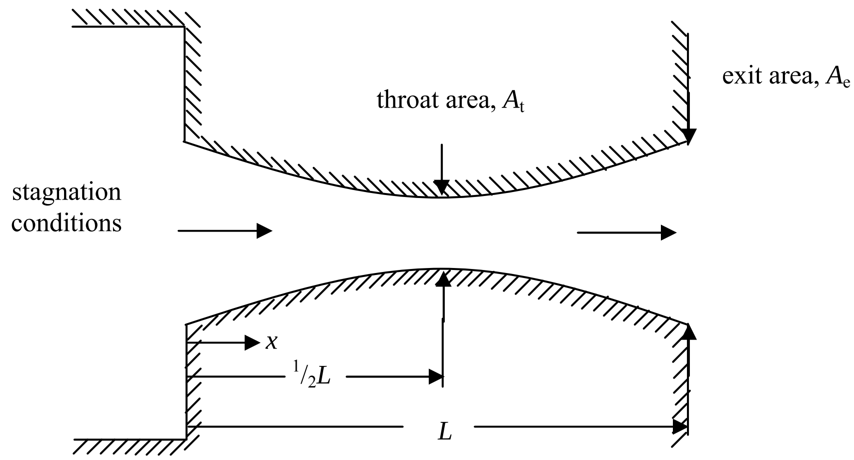
normalshock_03

A converging-diverging nozzle, with an exit to throat area ratio, A_e/A_t, of 1.633, is designed to operate with atmospheric pressure at the exit plane, $p_e = p_{atm}$.

a. Determine the range(s) of stagnation pressures for which the nozzle will be free from normal shocks.
b. If the stagnation pressure is $1.5p_{atm}$, at what position, x, will the normal shock occur?

The converging-diverging nozzle area, A, varies with position, x, as:

$$\frac{A(x)}{A_E} = \left(\frac{A_E}{A_T} - 1\right)\left(2\frac{x}{L} - 1\right)^2 + 1$$



Answer(s):

$$1 \le \frac{p_0}{p_{atm}} \le 1.11 \quad \text{and} \quad \frac{p_0}{p_{atm}} > 1.71$$

$x/L = 0.9306$

normalshock_04

An air stream approaches a normal shock at $Ma_1 = 2.64$. Upstream, $p_{01} = 3.00$ MPa (abs) and $\rho_1 = 1.65$ kg/m^3. Determine the downstream Mach number and temperature.

Answer(s):

$\therefore Ma_2 = 0.50$

$T_2 = 680$ K

normalshock_05

Air approaches a normal shock with $T_1 = 18$ °C, $p_1 = 101$ kPa (abs), and $V_1 = 766$ m/s. The temperature immediately downstream from the shock is $T_2 = 551$ K.

1. Determine the velocity immediately downstream from the shock.
2. Determine the pressure change across the shock.
3. Calculate the corresponding pressure change for a frictionless, shockless deceleration between the same speeds.

Answer(s):

$V_2 = 254.3$ m/s

$\Delta p = 4.73 \times 10^5$ Pa

$\Delta p_{\text{isentropic}} = 8.41 \times 10^5$ Pa

normalshock_06

Air flows through a converging-diverging nozzle, with $A_e/A_t = 3.5$ where $A_t = 500\ \text{mm}^2$. The upstream stagnation conditions are atmospheric; the back pressure is maintained by a vacuum system. Determine the range of back pressures for which a normal shock will occur within the nozzle and the corresponding mass flow rate.

Answer(s):

$0.331 < p_b/p_0 < 0.980$

$\dot{m}_{\text{choked}} = 0.119\ \text{kg/s}$

normalshock_07

A total pressure probe is inserted into a supersonic air flow. A shock wave forms just upstream of the impact hole. The probe measures a total pressure of 500 kPa (abs) and the stagnation temperature at the probe head is 227 °C. The static pressure upstream of the shock is measured with a wall tap to be 100 kPa (abs).

a. Determine the Mach number of the incoming flow.
b. Determine the velocity of the incoming flow.
c. Sketch the process on a *T*-*s* diagram.

Answer(s):

$\boxed{Ma_1 = 1.87}$

$\boxed{V_1 = 643.1 \text{ m/s}}$

normalshock_13

Air flows through a frictionless, adiabatic converging-diverging nozzle. The air in the reservoir feeding the nozzle has a pressure and temperature of 700 kPa and 500 K, respectively. The ratio of the nozzle exit to throat area is 11.91. A normal shock wave stands where the upstream Mach number is 3.0. Calculate the Mach number, the static temperature, and static pressure at the nozzle exit plane.

Answer(s):

$\text{Ma}_E = 0.15$

$T_E = 498$ K

$p_E = 226$ kPa (abs)

normalshock_15

A converging-diverging nozzle, with $A_e/A_t = 1.633$, is designed to operate with atmospheric pressure at the exit plane. Determine the range(s) of stagnation pressures for which the nozzle will be free from normal shocks.

Answer(s):

$p_{atm} \leq p_0 \leq 1.11 p_{atm}$ and $p_0 > 1.71\ p_{atm}$

normalshock_17

According to a newspaper article, at the center of a 12,600 lb_m "Daisy-Cutter" bomb explosion the overpressure in the air is approximately 1000 psi. Estimate:

a. the speed of the resulting shock wave into the surrounding air,
b. the wind speed following the shock wave,
c. the temperature after the shock wave has passed, and
d. the air density after the shock wave has passed.

Answer(s):

$\therefore T_2 = 6600\ ^\circ\text{R} = 6100\ ^\circ\text{R}$

$\therefore \rho_2 = 0.42\ \text{lb}_\text{m}/\text{ft}^3$

$\therefore V_{\substack{\text{shock,}\\ \text{w/r/t ground}}} = 8700\ \text{ft/s}$

$\therefore V_{\substack{\text{downstream wind,}\\ \text{w/r/t ground}}} = 7100\ \text{ft/s}$

normalshock_20

An automobile tire bursts sending a shock wave (assume this is a normal shock wave) propagating into the ambient air which has a pressure of p_1, sonic speed, c_1, and specific heat ratio, γ. If the pressure behind the shock is p_2 (roughly the inflated tire pressure), show that the speed of propagation of the shock, u_S, is given by:

$$u_S = c_1\sqrt{\frac{\gamma-1}{2\gamma}+\frac{p_2}{p_1}\frac{\gamma+1}{2\gamma}}$$

Calculate this speed if the temperature of the ambient air is 30 °C and the pressure ratio is $p_2/p_1 = 3.0$ (*e.g.* $p_1 = 14.7$ psia and $p_2 = 44.1$ psia).

Answer(s):

$u_S = 574.8$ m/s

normalshock_23

A supersonic aircraft flies at a Mach number of 2.7 at an altitude of 20 km. Air enters the engine inlet and is slowed isentropically to a Mach number of 1.3. A normal shock occurs at that location. The resulting flow is decelerated adiabatically, but not isentropically, further to a Mach number of 0.4. The final static pressure is 104 kPa (abs). Evaluate:

a. the stagnation temperature for the flow,
b. the pressure change, Δp, across the shock,
c. the final stagnation pressure, and
d. the total entropy change throughout the entire process.
e. Sketch the process on a Ts diagram.

Answer(s):

$T_0 = 533$ K

$\Delta p = 37.0$ kPa

$p_{04} = 116$ kPa (abs)

$\Delta s = 26.3$ J/(kg·K)

normalshock_24

The Mach number and temperature upstream of a shock wave are 2 and 7 °C, respectively. What is the air speed, relative to the shock wave, downstream of the shock wave?

Answer(s):

$V_2 = 252$ m/s