Chapter 10

THE FIRST LAW APPLIED TO STEADY FLOW PROCESSES

It is not the sun to overtake the moon, nor doth the night outstrip the day. They float each in an orbit.

– *The Holy Qur-ān*

In many engineering applications, devices such as turbines, pumps, compressors, heat exchangers and boilers are operated under steady flow conditions for long periods of time. A steady flow process is a process in which matter and energy flow in and out of an open system at steady rates. Moreover, an open system undergoing a steady flow process does not experience any change in the mass and energy of the system. Application of the first law of thermodynamics to steady flow processes is discussed in this chapter.

10.1 What is Steady?

The term steady implies **no change with time**. We say that a person is running at a steady speed of 5 km per hour, as shown in Figure 10.1, if the speed does not change with time.

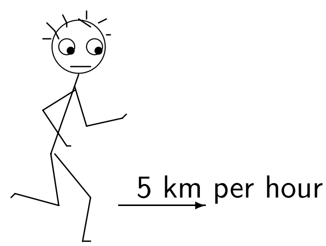


Figure 10.1 A person running at a steady speed of 5 km per hour.

10.2 What is a Steady Flow Process?

A steady flow process is one in which matter and energy flow steadily in and out of an open system. In a steady flow process, the properties of the flow remain unchanged with time, that is, the properties are frozen in time. But, the properties need not be the same in all points of the flow.

It is very common for a beginner to confuse the term steady with the term equilibrium. But, they are not the same. When a system is at a steady state, the properties at any point in the system are steady in time, but may vary from one point to another point. The temperature at the inlet, for example, may differ from that at the outlet. But, each temperature, whatever its value, remains constant in time in a steady flow process.

When a system is at an equilibrium state, the properties are steady in time and uniform in space. By properties being uniform in space, we mean that a property, such as pressure, has the same value at each and every point in the system.

An example of steady flow of water through a pipe is shown in Figure 10.2. Pressure measurements taken along the pipe at two different times of a day, shown in the figure, remain the same since the flow is steady. But, observe that the values of pressure vary along the pipe illustrating the nonuniform nature of the steady flow.

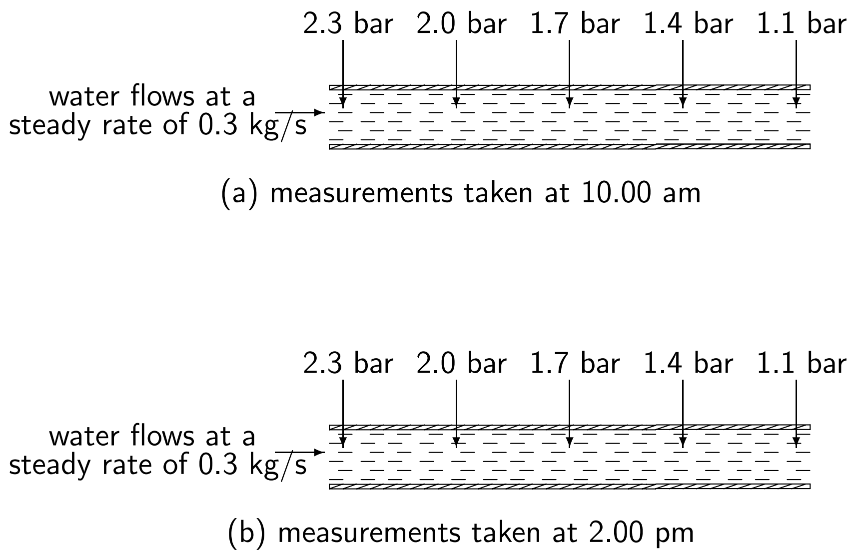


Figure 10.2 An example of steady flow.

10.3 Characteristics of a Steady Flow Process

A steady flow is one that remains unchanged with time, and therefore a steady flow has the following characteristics:

- Characteristic 1:

  No property at any given location within the system boundary changes with time. That also means, during an entire steady flow process, the total volume V_s of the system remains a constant, the total mass m_s

of the system remains a constant, and that the total energy content E_s of the system remains a constant.

- Characteristic 2:

  Since the system remains unchanged with time during a steady flow process, the system boundary also remains the same.

- Characteristic 3:

  No property at an inlet or at an exit to the open system changes with time. That means that during a steady flow process, the mass flow rate, the energy flow rate, pressure, temperature, specific (or molar) volume, specific (or molar) internal energy, specific (or molar) enthalpy, and the velocity of flow at an inlet or at an exit remain constant.

- Characteristic 4:

  Rates at which heat and work are transferred across the boundary of the system remain unchanged.

10.4 Mass Balance for a Steady Flow Process

Since a steady flow process can be considered as a special process experienced by the open system discussed in Chapter 9, we may start from the mass balance for open systems, which is given by (9.1). Characteristic 1 of the steady flow process is that the mass of the open system experiencing a steady flow process remains constant. This is achieved if the mass flow rate at the inlet equals the mass flow rate at the exit. Therefore, (9.1) reduces to

$$\boxed{\dot{m}_i = \dot{m}_e} \tag{10.1}$$

where the subscript i denotes the inlet and the subscript e denotes the exit.

10.5 Energy Balance for a Steady Flow Process

Since a steady flow process can be considered as a special process experienced by the open system discussed in Chapter 9, let us start with (9.8) which is the energy balance applicable to open systems. According to Characteristic 1 of the steady flow process, the total energy content E_s of the system remains constant during the process. Therefore

$$\frac{dE_s}{dt} = 0$$

According to Characteristic 2 of the steady flow process, the boundary remains unchanged with time, so that no boundary work is done during a steady flow process, and therefore

$$(\dot{W}_{boundary})_{in} = 0$$

According to Characteristic 3, all properties at the inlet and the exit of the system remain unchanged with time. Therefore, h, c and z are constants.

Applying all the above characteristics of a steady flow process to (9.8), we get

$$\begin{aligned} \dot{Q}_{in} + (\dot{W}_{shaft})_{in} \quad + \quad & \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz\right)_i \\ & - \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz\right)_e \; = \; 0 \end{aligned}$$

which may be organised as

$$\begin{aligned} \dot{Q}_{in} + (\dot{W}_s)_{in} \quad = \quad & \underbrace{\dot{m}_e \left(h_e + \frac{c_e^2}{2} + gz_e\right)}_{\text{for exit}} \\ & - \underbrace{\dot{m}_i \left(h_i + \frac{c_i^2}{2} + gz_i\right)}_{\text{for inlet}} \end{aligned} \tag{10.2}$$

where W_s is shaft work, and the subscripts i and e denotes the inlet and exit, respectively.

It is important to note that each of the rates in (10.2) is a constant for a steady flow process as pointed out in Characteristics 3 and 4 of steady flow processes.

Equation (10.1) states that $\dot{m}_i$ is the same as $\dot{m}_e$. Let us represent these two equal mass flow rates by the symbol $\dot{m}$, which can be considered as the constant mass flow rate through the steady flow process. Using the above in (10.2), we get the energy balance, that is the first law of thermodynamics applied to a steady flow process with a single inlet and a single exit, as

$$\boxed{\begin{aligned}\dot{Q}_{in} &+ (\dot{W}_s)_{in} \\ &= \dot{m}\left[h_e - h_i + \frac{c_e^2 - c_i^2}{2} + g\,(z_e - z_i]\right]\end{aligned}} \qquad (10.3)$$

which is the **steady flow energy equation** (abbreviated to SFEE) applicable to a single-stream steady flow process. The rate at which heat enters the system is constant at $\dot{Q}_{in}$. The rate at which shaft work enters the system is constant at $(\dot{W}_s)_{in}$. The mass flow rates of the single stream entering and leaving the system are constant at $\dot{m}$. The specific enthalpy of the stream at the inlet, the velocity of the stream at the inlet and the elevation of the inlet are constant at h_i, c_i, and z_i, respectively, and those at the exit are constant at h_e, c_e, and z_e, respectively. The acceleration due to gravity is denoted by g.

For a multiple-stream steady flow process, that is a system with several inlets and exits for mass to flow, the steady flow energy equation (SFEE) becomes

$$\begin{aligned}\dot{Q}_{in} + (\dot{W}_s)_{in} &= \left[\dot{m}_{e1}\left(h_{e1} + \frac{c_{e1}^2}{2} + gz_{e1}\right)\right] \\ &\quad + \left[\dot{m}_{e2}\left(h_{e2} + \frac{c_{e2}^2}{2} + gz_{e2}\right)\right] + \cdots \\ &\quad - \left[\dot{m}_{i1}\left(h_{i1} + \frac{c_{i1}^2}{2} + gz_{i1}\right)\right] \\ &\quad - \left[\dot{m}_{i2}\left(h_{i2} + \frac{c_{i2}^2}{2} + gz_{i2}\right)\right] - \cdots \end{aligned} \qquad (10.4)$$

which is solved together with the mass balance for a multiple-stream steady flow process,

$$\dot{m}_{e1} + \dot{m}_{e1} + \cdots = \dot{m}_{i1} + \dot{m}_{i2} + \cdots \tag{10.5}$$

where the subscripts $e1$, $e2$, $\cdots$ denote exit 1, exit 2, and so on, respectively, and the subscripts $i1$, $i2$, $\cdots$ denote inlet 1, inlet 2, and so on, respectively.

10.6 Steady Flow Engineering Devices

Many engineering devices operate essentially under unchanged conditions for long periods. For example, the industrial appliances such as turbines, compressors, heat exchangers and pumps may operate nonstop at steady state for months before they are shut down for maintenance. In this section, we will deal with devices such as nozzles, turbines, compressors, heat exchangers, boilers, and condensers. The emphasis will, however, be on the overall functions of the devices, and the steady flow energy equation will be applied to these devices treating them more or less like black boxes.

Nozzles & Diffusers

Nozzles and diffusers are properly shaped ducts which are used to increase or decrease the speed of the fluid flowing through it. Schematics of a typical nozzle and a typical diffuser are shown in Figure 10.3. Nozzles are used for various applications such as to increase the speed of water through a garden hose, and to increase the speed of the gases leaving the jet engine or rocket. Diffusers are used to slow down a fluid flowing at high speeds, such as at the entrance of a jet engine.

Since no shaft work is involved in a nozzle or a diffuser, and since the potential energy difference across a nozzle or a diffuser is usually negligible, the steady flow energy equation (10.3) for flow through a nozzle or a diffuser becomes

$$\dot{Q}_{in} = \dot{m}\left(h_e - h_i + \frac{c_e^2 - c_i^2}{2}\right) \tag{10.6}$$

Figure 10.3 Schematics of a nozzle and a diffuser.

The flow through nozzles and diffusers are often considered adiabatic, so that the rate of heat transfer is neglected. Therefore (10.6) reduces to

$$\frac{c_e^2 - c_i^2}{2} = h_i - h_e \tag{10.7}$$

for adiabatic nozzles and diffusers. It can be clearly seen in (10.7) that an increase in the speed of the flow is accompanied by a decrease in its enthalpy, as in the case of flow through an adiabatic nozzle. And, a decrease in the speed of the flow is accompanied by an increase in its enthalpy, as in the case of flow through an adiabatic diffuser.

Turbines

A turbine is a device with rows of blades mounted on a shaft which could be rotated about its axis (see Figure 1.1). In some water turbines used in hydroelectric power stations, water at high velocity is directed at the blades of the turbine to set the turbine shaft in rotation. The work delivered by the rotating shaft drives an electric generator to produce electrical energy. In steam turbines, steam at high pressure and temperature enters a turbine, sets the turbine shaft in rotation, and leaves at low pressure and temperature. In gas turbines, gaseous products of combustion at high pressure and temperature set the turbine shaft in rotation. The rotating shaft of a turbine is not always used for electric power generation. It is also an essential part of a jet engine in an aircraft which generates the thrust required to propel the aircraft. The schematic of a turbine is shown in Figure 10.4.

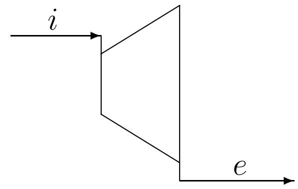


Figure 10.4 Schematic of a turbine.

Since the fluid flowing through a turbine usually experiences negligible change in elevation, the potential energy term is neglected. Work always leaves the turbine, and therefore the $(\dot{W}_s)_{in}$ term in (10.3) is negative. The steady flow energy equation for flow through a turbine may therefore be written as

$$\dot{Q}_{in} - (\dot{W}_s)_{out} = \dot{m}\left(h_e - h_i + \frac{c_e^2 - c_i^2}{2}\right) \tag{10.8}$$

The fluid velocities encountered in most turbines are large, and the fluid experiences a significant change in its kinetic energy. However, if this change is small compared to the change in enthalpy then the change in kinetic energy may be neglected. If the fluid flowing through the turbine undergoes an adiabatic process, which is usually the case, then $\dot{Q}_{in} = 0$.

Under such conditions, (10.8) reduces to

$$(\dot{W}_s)_{out} = \dot{m}\,(h_i - h_e) \tag{10.9}$$

which clearly shows that the shaft work delivered by an adiabatic turbine is derived from the enthalpy loss by the fluid flowing through the turbine.

Compressors

A compressor is a device used to increase the pressure of a gas flowing through it. The rotating type compressor functions in a manner opposite to a turbine. To rotate the shaft of a compressor, work must be supplied from an external source such as a rotating turbine shaft. The blades that are mounted on the shaft of the compressor are so shaped that, when the compressor shaft rotates, the pressure of the fluid flowing through the compressor increases. The rotating type compressors are used to raise the pressure of the air flowing through it in the electricity generation plants and in the jet engines. In a reciprocating type compressor, a piston moves within the cylinder, and the work needed to move the piston is generally supplied by the electricity obtained from a wall socket. Household refrigerators use the reciprocating type of compressors to raise the pressure of the refrigerant flowing through them. A schematic of a compressor is shown in Figure 10.5.

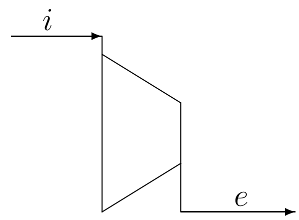


Figure 10.5 Schematic of a compressor.

The potential energy difference across a compressor is usually neglected, and the steady flow energy equation for flow through it becomes

$$\dot{Q}_{in} + (\dot{W}_s)_{in} = \dot{m}\left(h_e - h_i + \frac{c_e^2 - c_i^2}{2}\right) \tag{10.10}$$

A pump works like a compressor except that it handles liquids instead of gases. Fans and blowers are compressors which impart a very small rise in the pressure of the fluid flowing through them, and are used mainly to circulate air. Equation (10.10) may be used to describe the flows through pumps, fans and blowers.

The velocities involved in these devices are usually small to cause a significant change in kinetic energy, and often the change in kinetic energy term is neglected. If the compressor, pump, fan or blower is operated under adiabatic conditions, then $\dot{Q}_{in} = 0$.

Under such conditions, (10.10) reduces to

$$(\dot{W}_s)_{in} = \dot{m}\,(h_e - h_i) \tag{10.11}$$

which clearly shows that the shaft work provided to an adiabatic compressor, pump, fan or blower is used to increase the enthalpy of the fluid flowing through.

Throttling Valves

A throttling valve is a device used to cause a pressure drop in a flowing fluid. It does not involve any work. The drop in pressure is attained by placing an obstacle such as a partially open valve, porous plug or a capillary tube in the path of the flowing fluid. The pressure drop in the fluid is usually accompanied by a drop in temperature, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications where a drop in the temperature of the working fluid is essential. A schematic of a throttling valve is shown in Figure 10.6.

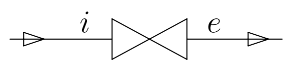


Figure 10.6 Schematic of a throttling valve.

Throttling valves are compact devices, and the flow through them is effectively adiabatic. There is no shaft work involved. The change in potential energy is neglected. The steady flow energy equation applied to flow through an adiabatic throttling valve becomes

$$h_e + \frac{c_e^2}{2} = h_i + \frac{c_i^2}{2} \tag{10.12}$$

Since the kinetic energy in many cases is insignificant when compared to the enthalpy, the kinetic energy terms are neglected. So that Equation (10.12) becomes

$$h_e \approx h_i \tag{10.13}$$

which shows the enthalpies at the inlet and the exit of a throttling valve are nearly the same.

If the enthalpy terms in (10.13) are expanded using $(u + Pv)$, we get

$$u_e + P_e\, v_e = u_i + P_i\, v_i$$

which means that the summation of internal energy u and the flow work Pv remains constant in a flow through a throttling valve.

If the flow work, Pv, increases during throttling, then internal energy, u, will decrease, which often means a decrease in temperature. On the contrary, if Pv decreases then u will increase, resulting in probable temperature increase. It means that the properties such as P, T and v of the fluid flowing through the throttling valve may change even though the enthalpy remains unchanged during throttling.

However, if the behaviour of the working fluid approximates that of an ideal gas then no change in enthalpy means no change in temperature as well.

Mixing Chambers

Mixing chamber refers to an arrangement where two or more fluid streams are mixed to form one single fluid stream as shown in the schematic in Figure 10.7. Mixing chambers are very common engineering applications in process industries.

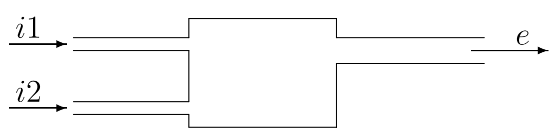


Figure 10.7 Schematic of a mixing chamber.

Since a mixing chamber has more than one inlet, we use the steady flow energy equation given by (10.4) to describe the flow through a mixing chamber. For the mixing chamber of Figure 10.7 with two inlets and one exit, (10.4) becomes

$$\dot{Q}_{in} = \dot{m}_e h_e - (\dot{m}_{i1} h_{i1} + \dot{m}_{i2} h_{i2}) \tag{10.14}$$

Note that there is no shaft work in a mixing chamber, and the changes in kinetic and potential energies of the streams are usually neglected.

For the conservation of mass across the mixing chamber of Figure 10.7, we can write

$$\dot{m}_e = \dot{m}_{i1} + \dot{m}_{i2}$$

which transforms (10.14) to

$$\dot{Q}_{in} = \dot{m}_{i1}(h_e - h_{i1}) + \dot{m}_{i2}(h_e - h_{i2}) \tag{10.15}$$

Mixing chambers are usually well insulated, so that the process can be treated as adiabatic. For an adiabatic mixing chamber, (10.15) reduces to

$$\dot{m}_{i1}(h_e - h_{i1}) = \dot{m}_{i2}(h_{i2} - h_e) \tag{10.16}$$

Heat Exchangers

In the industries, there is often a need to cool a hot fluid stream before it is let out into the environment. The heat removed from cooling of a hot fluid can be used to heat another fluid that has to be heated up. This can be achieved in a heat exchanger, which in general is a device where a hot fluid stream exchanges heat with a cold fluid stream without mixing with each other. The simplest type is the double-pipe heat exchanger which has two concentric pipes of different diameters. One fluid flows in the inner pipe and the other in the annular space between the two pipes. The schematic of a double-pipe heat exchanger is shown in Figure 10.8.

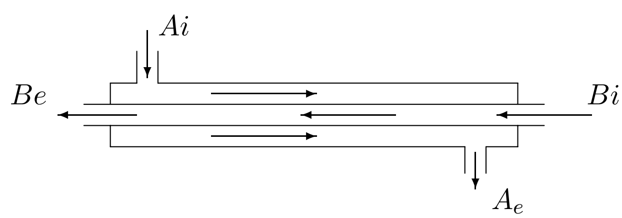


Figure 10.8 Schematic of a double-pipe heat exchanger.

Since a heat exchanger has two inlets and two exits, we use (10.4). There is no work transfer, and the changes in kinetic and potential energies are neglected. Therefore, (10.4) reduces to

$$\dot{Q}_{in} = [\dot{m}_{Ae}h_{Ae} + \dot{m}_{Be}h_{Be}] - [\dot{m}_{Ai}h_{Ai} + \dot{m}_{Bi}h_{Bi}] \tag{10.17}$$

Since the mass flow rate of fluid A is the same at the inlet and at the exit, $\dot{m}_{Ae} = \dot{m}_{Ai}$, which may be represented by $\dot{m}_A$. Since the mass flow rate of fluid B is the same at the inlet and at the exit, $\dot{m}_{Be} = \dot{m}_{Bi}$, which may be represented by $\dot{m}_B$ Thus, (10.17) becomes

$$\dot{Q}_{in} = \dot{m}_A \left[h_{Ae} - h_{Ai}\right] + \dot{m}_B \left[h_{Be} - h_{Bi}\right] \tag{10.18}$$

Where a heat exchanger is insulated, it is adiabatic and the heat transfer term may be neglected. So that (10.18) reduces to

$$\frac{\dot{m}_{fluid\ A}}{\dot{m}_{fluid\ B}} = \frac{(h_e - h_i)_{fluid\ B}}{(h_i - h_e)_{fluid\ A}} \tag{10.19}$$

Boilers and Condensers

A liquid is converted into vapour in a boiler by supplying heat to it. A boiler, for example, is used to heat water at room temperature to its boiling temperature so that water may be converted into steam. Heat may be supplied to the boiler by burning a fuel in the boiler. In a condenser, a vapour is condensed to liquid by removing heat from it. Schematics of a boiler and a condenser are shown in Figure 10.9.

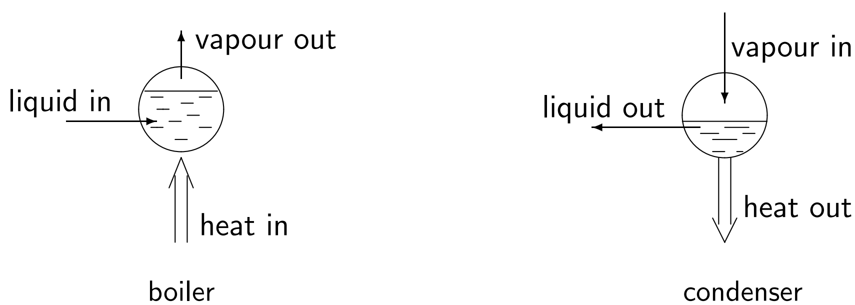


Figure 10.9 Schematics of a boiler and a condenser.

There is no shaft work involved in a boiler or a condenser. The potential and kinetic changes across these devices are negligible in comparison to the change in enthalpy. So that the steady flow energy equation for flow through a boiler becomes

$$\dot{Q}_{in} = \dot{m}\left(h_e - h_i\right) \tag{10.20}$$

and the steady flow energy equation for flow through a condenser becomes

$$\dot{Q}_{out} = \dot{m}\left(h_i - h_e\right) \tag{10.21}$$

10.7 Worked Examples

Example 10.1

Gases produced during the combustion of a fuel-air mixture, enter a nozzle at 200 kPa, 150°C and 20 m/s and leave the nozzle at 100 kPa and 100°C. The exit area of the nozzle is 0.03 m^2. Assume that these gases behave like an ideal gas with C_p = 1.15 kJ/kg · K and γ = 1.3, and that the flow of gases through the nozzle is steady and adiabatic. Determine (i) the exit velocity and (ii) the mass flow rate of the gases.

Solution to Example 10.1

(i) Determination of the exit velocity

The given flow may be satisfactorily described by (10.7). Since the behaviour of the gases is approximated to that of an ideal gas with constant C_p, (10.7) can be rewritten as

$$\frac{c_e^2 - c_i^2}{2} = C_p\left(T_i - T_e\right) \tag{10.22}$$

Substituting the values given in the problem statement in (10.22), we get

$$\begin{aligned}\frac{c_e^2}{2} &= \frac{20^2}{2}\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(1.15\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) \times (423\ \text{K} - 373\ \text{K}) \\ &= 200\left(\frac{\text{m}}{\text{s}}\right)^2 + 57.5\left(\frac{\text{kJ}}{\text{kg}}\right).\end{aligned}$$

We cannot add a quantity in kJ/kg to a quantity in $(\text{m/s})^2$. But, we can add a quantity in J/kg to a quantity in $(\text{m/s})^2$ since they are equivalent as shown below:

$$\frac{\text{J}}{\text{kg}} = \frac{\text{N}\cdot\text{m}}{\text{kg}} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\,\frac{\text{m}}{\text{kg}} = \left(\frac{\text{m}}{\text{s}}\right)^2$$

Therefore,

$$\begin{aligned} c_e &= \sqrt{2\times\left[200\ \left(\frac{\text{m}}{\text{s}}\right)^2 + 57.5\times 1000\ \left(\frac{\text{J}}{\text{kg}}\right)\right]} \\ &= 339.7\ \text{m/s} \end{aligned}$$

Note that the speed of the gases flowing through the nozzle is increased from 20 m/s to 339.7 m/s, which is achieved at the cost of the reduction in the gas pressure from 200 kPa to 100 kPa.

(ii) Determination of the mass flow rate of the gases

Assume that the gas flows perpendicular to a cross sectional area A at a uniform speed c and at a uniform density ρ. The mass flow rate of the gases through the given cross-section is then

$$\dot{m} = A\,\rho\,c = A\,c/v \tag{10.23}$$

where v is the specific volume.

Since we assume ideal gas behaviour, the ideal gas equation of state may be used to express v as

$$v = \frac{RT}{P}$$

Thus, the mass flow rate of an ideal gas through the cross-sectional area A can be written as

$$\dot{m} = \frac{A\,c\,P}{RT} \tag{10.24}$$

As we know the exit area, exit pressure, exit velocity and exit temperature, and the gas constant R, calculated using $R = (\gamma-1)\,C_p/\gamma = 0.265$ kJ/kg·K,

$$\dot{m} = \frac{(0.03\ \text{m}^2)\times(339.7\ \text{m/s})\times(100\ \text{kPa})}{(0.265\ \text{kJ/kg}\cdot\text{K})\times(373\ \text{K})} = 10.31\ \text{kg/s}$$

Example 10.2

Rework **Example 10.1** assuming that the expansion of the gases flowing through the nozzle from 200 kPa, 150°C and 20 m/s at the inlet to 100 kPa at the exit takes place quasistatically.

Solution to Example 10.2

(i) Determination of the exit velocity

Since the flow through the nozzle is assumed to be a quasistatic adiabatic flow of an ideal gas, P and T of the flow can be related using (7.31), which gives

$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{(\gamma-1)/\gamma} = 423 \text{ K} \times \left(\frac{100}{200}\right)^{(1.3-1)/1.3} = 360.5 \text{ K}$$

Substituting the values of c_i, T_i and T_e in (10.22), we get

$$\begin{aligned} c_e &= \sqrt{2 \times \left[200 \left(\frac{\text{m}}{\text{s}}\right)^2 + 1.15 \times (423 - 360.5) \times 1000 \left(\frac{\text{J}}{\text{kg}}\right)\right]} \\ &= 379.7 \text{ m/s} \end{aligned}$$

Note that the speed of the gases flowing through the nozzle, expanding from 200 kPa to 100 kPa, is increased from 20 m/s to 379.7 m/s when the flow is assumed to be quasistatic.

(ii) Determination of the mass flow rate of the gases

The mass flow rate of the gases through the nozzle is given by

$$\dot{m} = \frac{(0.03 \text{ m}^2) \times (379.7 \text{ m/s}) \times (100 \text{ kPa})}{(0.265 \text{ kJ/kg} \cdot \text{K}) \times (360.5 \text{ K})} = 11.92 \text{ kg/s}$$

`Student:` Teacher, for an expansion or a compression process to be quasistatic, it must take place under fully restrained condition. Is that correct?

`Teacher:` Yes, that is correct.

`Student:` It is stated in **Example 10.2** that the expansion of the flow through the nozzle is assumed to be quasistatic. How could that be when there is nothing to restrain the expansion of the gases flowing through the nozzle?

`Teacher:` Yes, you are right about that. The flow through a nozzle is far from quasistatic. However, we assume the flow to be quasistatic in order to determine the maximum possible speed that could be attained by the gases flowing through the nozzle. Observe that the speed of the gases at the exit is 379.7 m/s under quasistatic adiabatic condition, which sets the maximum possible speed attainable by the gases flowing through the nozzle under the same inlet conditions and exit pressure.

`Student:` Oh... I see.

`Teacher:` It would be interesting to take look at the ratio between the actual kinetic energy change per kg of flow and the ideal kinetic energy change per kg of flow achievable with quasistatic adiabatic flow under the same pressure difference between the inlet and the outlet of the nozzle and the same inlet temperature.

$$\begin{aligned}\text{The required ratio} &= \frac{[(c_e^2 - c_i^2)/2]_{actual}}{[(c_e^2 - c_i^2)/2]_{ideal}} \\ &= \frac{(339.7^2 - 20^2)}{(379.7^2 - 20^2)} \\ &= 0.8\end{aligned}$$

That is, the nozzle of **Example 10.1** achieves only about 80% of the kinetic energy increase per kg of flow achievable under ideal flow conditions. This way we determine the efficiency at which a nozzle operates.

`Student:` Okay, I see now why the flow is assumed to be quasistatic in **Example 10.2**. However, I have a question. How do you know that a quasistatic flow gives the maximum possible speed attainable by the gas flow through the nozzle?

`Teacher:` You are wrong. It is not the quasistatic flow, but the quasistatic adiabatic flow that gives the maximum possible speed attainable by the gas flow through the nozzle?

`Student:` Okay, Teacher. I still have a question. How do you know that a quasistatic adiabatic flow gives the maximum possible speed attainable by the gas flow through the nozzle?

`Teacher:` When learning the second law, you will see that it could be proved that a quasistatic flow sets the limit for the best performance that could be expected of an engineering device operated under adiabatic conditions.

Example 10.3

Rework **Example 10.1** with steam flowing through the nozzle.

Solution to Example 10.3

(i) The given flow can be described by (10.7), of which h_i and h_e are the specific enthalpies of the steam at the inlet (2 bar and 150°C) and at the exit (1 bar and 100°C). From a Steam Table, we find that $h_i = 2770$ kJ/kg and $h_e = 2676$ kJ/kg. Substituting these values in (10.7) along with $c_i = 20$ m/s, we get

$$\begin{aligned} c_e &= \sqrt{2 \times \left[200 \left(\frac{\text{m}}{\text{s}}\right)^2 + (2770 - 2676) \times 1000 \left(\frac{\text{J}}{\text{kg}}\right)\right]} \\ &= 434 \text{ m/s} \end{aligned}$$

(ii) The mass flow rate of the steam may be calculated using (10.23) of which the specific volume v cannot be expressed as RT/P since the behaviour of steam may not be approximated by the ideal gas behaviour. However, it is not a problem because v for steam can be obtained from a Steam Table. We know the cross-sectional area and the speed of steam at the exit. We can get v at the exit at 1 bar and 100°C from the Steam Table as 1.696 m^3/kg. Substituting these values in (10.23), we get

$$\dot{m} = \frac{(0.03 \text{ m}^2) \times (434 \text{ m/s})}{1.696 \text{ m}^3/\text{kg}} = 7.68 \text{ kg/s}$$

Example 10.4

Steam entering a nozzle at 7 bar and 250°C with a velocity of 10 m/s, leaves it at 3 bar 200°C with a velocity of 262 m/s. Determine the heat lost by the steam flowing through the nozzle. If the mass flow rate of the steam flowing through the nozzle is 2.5 kg/s, determine the inlet area of the nozzle.

Solution to Example 10.4

The given flow can be described by (10.6), of which h_i and h_e are the specific enthalpies of the steam at the inlet (7 bar and 250°C) and at the exit (3 bar and 200°C). From a Steam Table, we find that $h_i = 2955$ kJ/kg and $h_e = 2866$ kJ/kg. Substituting these values in (10.6) along with $c_i = 10$ m/s and $c_e = 262$ m/s, we get

$$\begin{aligned} \dot{Q}_{in} &= 2.5\ \frac{\text{kg}}{\text{s}} \times \left[(2866 - 2955) \times 1000 \left(\frac{\text{J}}{\text{kg}}\right) + \frac{262^2 - 10^2}{2} \left(\frac{\text{m}}{\text{s}}\right)^2 \right] \\ &= -136.8\ \text{kJ} \end{aligned}$$

The heat lost by the steam flowing through the nozzle is 136.8 kJ.

The inlet area of the nozzle can be calculated using (10.23) as follows:

$$A_i = \frac{\dot{m}\, v_i}{c_i}$$

where $\dot{m} = 2.5$ kg/s, $c_i = 10$ m/s and $v_i = v$ at 7 bar and 250°C = 0.3364 m^3/kg. The inlet area of the nozzle is therefore 0.0841 m^2.

Example 10.5

Air ($C_p = 1.005$ kJ/kg·K; $\gamma = 1.4$) enters an adiabatic diffuser at 85 kPa and 250 K at a steady speed of 265 m/s and leaves it at 15 m/s. Assuming ideal gas behaviour for air with constant specific heats, determine the pressure and temperature of the air at the diffuser exit.

Solution to Example 10.5

Equation (10.7) applied to the air flow through the adiabatic diffuser assuming ideal gas behaviour gives (10.22). Substituting the given numerical values known from the problem statement in (10.22), we get

$$T_e = 250 \text{ K} - \left(\frac{15^2 - 265^2}{2 \times 1005} \right) \frac{(\text{m/s})^2}{\text{J/kg} \cdot \text{K}} = 284.8 \text{ K} \tag{10.25}$$

which gives the temperature of air at the diffuser exit.

To determine the pressure at the exit, there is not enough data provided. However, if we assume quasistatic flow through the diffuser then, since the flow is adiabatic and since ideal gas behaviour is assumed, P and T of the flow can be related using (7.31), which gives

$$P_e = P_i \left(\frac{T_e}{T_i} \right)^{\gamma/(\gamma-1)} = 85 \text{ kPa} \times \left(\frac{284.8}{250} \right)^{1.4/(1.4-1)} = 134 \text{ kPa}$$

which gives the pressure of air at the diffuser exit under ideal conditions.

That is, under quasistatic adiabatic flow conditions, the air flowing through the diffuser is compressed from 85 kPa to 134 kPa, which is achieved at the cost of the reduction in the air speed from 265 m/s to 15 m/s.

Example 10.6

A mixture of gases enter a nozzle at 2.5 bar and 237°C with a speed of 20 m/s. The inlet diameter of the nozzle is 0.45 m. We are required to achieve an exit velocity of 340 m/s. Assuming quasistatic flow conditions through the nozzle, determine (i) the pressure that should be maintained at the nozzle exit and (ii) the exit diameter. Take C_p = 1.15 kJ/kg·K and γ = 1.3 for the gases. Assume ideal gas behaviour and steady adiabatic flow through the nozzle.

Solution to Example 10.6

(i) Since the flow is assumed to be a quasistatic adiabatic flow of an ideal gas

(7.31) could be used to determine the pressure at the nozzle exit as follows:

$$P_e = 2.5 \text{ bar} \times \left(\frac{T_e}{510}\right)^{1.3/(1.3-1)} \tag{10.26}$$

where the exit temperature T_e is unknown.

For the flow of gases, assumed to behave as an ideal gas, through an adiabatic nozzle, (10.7) is applicable. Substituting the values known from the problem statement in (10.7), we get

$$1.15 \left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(T_e - 510 \text{ K}) + \left(\frac{340^2 - 20^2}{2}\right)\left(\frac{\text{m}}{\text{s}}\right)^2 = 0$$

which gives $T_e = 460$ K.

Substituting this value of T_e in (10.26) we get $P_e = 1.6$ bar. That is, an exit pressure of about 1.6 bar should be maintained for the combustion gases to be able to reach the speed of 340 m/s^2 at the exit.

(ii) To determine the exit area, we use (10.24) as follows:

$$\dot{m} = \left(\frac{A\,c\,P}{RT}\right)_{inlet} = \left(\frac{A\,c\,P}{RT}\right)_{exit}$$

which gives

$$A_e = A_i\left(\frac{P_i}{P_e}\right)\left(\frac{c_i}{c_e}\right)\left(\frac{T_e}{T_i}\right) = A_i\left(\frac{2.5}{1.6}\right)\left(\frac{20}{340}\right)\left(\frac{460}{510}\right) = 0.083A_i$$

Since $A_i = (\pi/4)(0.45 \text{ m})^2 = 0.159$ m^2, the exit area of the nozzle is about 0.013 m^2, and the exit diameter of the nozzle is about 0.13 m.

Example 10.7

A gas turbine is operated with gases ($C_p =$ 0.992 kJ/kg·K and $\gamma = 1.29$) entering it at 10 bar and 1025°C, and leaving it at 1 bar and 550°C. Assuming adiabatic flow through the turbine, calculate the power output of the turbine (in MW) for each kg per second of gases flowing through the turbine.

If 120 MW of power is to be produced by the turbine, determine the mass flow rate of gases flowing through the turbine.

Solution to Example 10.7

Equation (10.9) can be used to describe the behaviour of the gases flowing through the adiabatic turbine for which $\dot{Q}_{in} = 0$. Since ideal gas behaviour is assumed (10.9) becomes

$$(\dot{W}_s)_{out} = \dot{m}\, C_p\, (T_i - T_e)$$

Substituting the known numerical values in the above equation, we get

$$(\dot{W}_s)_{out} = \dot{m} \times 0.992\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \times (1025 - 550)\ \text{K} = \dot{m} \times 471.2\ \text{kJ/kg}$$

which gives

$$\frac{(\dot{W}_s)_{out}}{\dot{m}} = 471.2\ \frac{\text{kJ}}{\text{kg}} = 471.2\ \frac{\text{kJ/s}}{\text{kg/s}} = 471.2\ \frac{\text{kW}}{\text{kg/s}} = 0.4712\ \frac{\text{MW}}{\text{kg/s}}$$

The power output of the turbine is therefore 0.4712 MW for each kg per second of gases flowing through the turbine.

Mass flow rate of gases required to produce 120 MW of power is calculated as follows:

$$\dot{m} = \frac{120\ \text{MW}}{0.4712\ \text{MW/kg/s}} = 254.7\ \text{kg/s}$$

Example 10.8

Rework **Example 10.7** under quasistatic adiabatic conditions, where the gases are supplied to the turbine at 10 bar and 1025°C, and they leave it at 1 bar.

Solution to Example 10.8

Ideal gas flow through the turbine under quasistatic adiabatic condition can be described by (7.31) so that

$$T_e = (1025 + 273)\ \text{K} \times \left(\frac{1}{10}\right)^{(1.29-1)/1.29} = 773.5\ \text{K} = 500.5^{\circ}\text{C}$$

Therefore

$$\frac{(\dot{W}_s)_{out}}{\dot{m}} = 0.992 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \times (1025 - 500.5)\ \text{K}$$
$$= 520.3\ \text{kJ/kg}$$

which gives that the power produced by the turbine under quasistatic adiabatic flow condition is 0.5203 MW for each kg per second of gases flowing through the turbine.

Mass flow rate of gases required to produce 120 MW of power is calculated as follows:

$$\dot{m} = \frac{120\ \text{MW}}{0.5203\ \text{MW/kg/s}} = 230.6\ \text{kg/s}$$

Observe that the turbine operating under quasistatic adiabatic condition produces more power, as in **Example 10.8**, which is 0.5203 MW, than the power produced by the turbine working under adiabatic but non-quasistatic condition, as in **Example 10.7**, for the same inlet condition and the exit pressure. The efficiency of the turbine in **Example 10.7**, can therefore be calculated as 0.4712/0.5203 = 90.6%.

Also, observe that the mass flow rate of the gas required to produce the same power output from the turbine operating under the same inlet condition and the exit pressure is less for the quasistatic adiabatic flow through the turbine, as in **Example 10.8**, than for the non-quasistatic adiabatic flow through the turbine, as in **Example 10.7**.

Example 10.9

A steam turbine producing 55 MW power is fed with steam at 70 bar and 500°C. Steam leaves the turbine at 0.08 bar with a dryness fraction of 0.90. Determine the mass flow rate of steam through the adiabatic turbine.

Solution to Example 10.9

Equation (10.9) can be used to describe the behaviour of steam flowing through the adiabatic turbine, where $(\dot{W}_s)_{out} = 55$ MW, and h_i is the enthalpy at 70 bar and 500°C and h_e is the enthalpy at 0.08 bar for a dryness fraction of 0.90. From a Superheated Steam Table, we find that $h_i = 3410$ kJ/kg. From a Saturated Water and Steam Table, we find that

$$\begin{aligned} h_i &= h_f + x\,(h_g - h_f) \quad \text{at 0.08 bar and } x = 0.90 \\ &= 174 \text{ kJ/kg} + 0.90\,(2576 - 174) \text{ kJ/kg} = 2336 \text{ kJ/kg} \end{aligned}$$

Substituting the known numerical values in (10.9), we get

$$55 \times 10^3 \,\frac{\text{kJ}}{\text{s}} = \dot{m} \times (3410 - 2336)\,\frac{\text{kJ}}{\text{kg}}$$

which gives $\dot{m}$, which is the mass flow rate of steam through the adiabatic turbine, as 51.2 kg/s.

Example 10.10

A steam turbine is fed with 53 kg/s of high pressure steam at 70 bar and 500°C and with 10 kg/s of low pressure steam at 6 bar and 250°C. The steam leaving the turbine is at 0.07 bar with the dryness fraction of 0.92. Determine the power produced by the steam turbine considering the fact that the heat loses from the turbine is equivalent to 2% of the power production.

Solution to Example 10.10

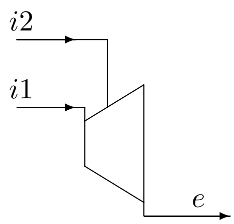
The schematic of the steam turbine given is shown in Figure 10.10. Since the turbine has two inlets and one exit, we must use the steady flow energy equation applied for a multiple-stream steady flow process given by (10.4). Neglecting the changes in potential and kinetic energies reduces (10.4) applied to the given system to

$$(\dot{Q})_{in} + (\dot{W}_s)_{in} = \dot{m}_e\, h_e - \dot{m}_{i1}\, h_{i1} - \dot{m}_{i2}\, h_{i2}$$

Since the power output is positive for a turbine and since heat is lost to the surroundings, the above equation can be rewritten as

$$(\dot{Q})_{out} + (\dot{W}_s)_{out} = \dot{m}_{i1}\, h_{i1} + \dot{m}_{i2}\, h_{i2} - \dot{m}_e\, h_e \qquad (10.27)$$

where $(\dot{Q})_{out}$ is given as 2% of $(\dot{W}_s)_{out}$.



$i1$ is high pressure steam at 70 bar and 500°C
$i2$ is low pressure steam at 6 bar and 250°C
e is steam at 0.07 bar and x = 0.92

Figure 10.10 Schematic of the turbine of Example 10.10.

From a Superheated Steam Table, we can find that the enthalpy of the high pressure steam h_{i1} = 3410 kJ/kg and the enthalpy of the low pressure steam h_{i2} = 2958 kJ/kg. From a Saturated Water and Steam Table, we can find that the enthalpy of the steam leaving the turbine h_e = 163 kJ/kg + 0.92 × 2409 kJ/kg = 2379 kJ/kg. The mass flow rates are given as $\dot{m}_{i1}$ = 53 kg/s and $\dot{m}_{i2}$ = 10 kg/s.

The mass balance applied for a multiple-stream steady flow process is given by (10.5), which for the given system becomes

$$\dot{m}_e = \dot{m}_{i1} + \dot{m}_{i2} = (53 + 10) \text{ kg/s} = 63 \text{ kg/s}$$

Substituting the known numerical values in (10.27), we get

$$\begin{aligned} 0.02 \times (\dot{W})_{out} + (\dot{W}_s)_{out} &= (53 \times 3410 + 10 \times 2958 - 63 \times 2379) \text{ kJ/s} \\ &= 60433 \text{ kW} \end{aligned}$$

which gives

$$(\dot{W})_{out} = \frac{60433 \text{ kW}}{1.02} = 59248 \text{ kW} = 59.25 \text{ MW}$$

That is, the steam turbine power output is 59.25 MW, and the heat lost to the surroundings is about 1.18 MJ/s.

Example 10.11

Air at 100 kPa and 300 K with a mass flow rate of 0.05 kg/s is to be compressed to 800 kPa using any one of the following methods:

Method 1: Quasistatic adiabatic compression from 100 kPa to 800 kPa in a single compressor.

Method 2: Quasistatic adiabatic compression from 100 kPa to 250 kPa in one compressor followed by quasistatic adiabatic compression from 250 kPa to 800 kPa in a second compressor.

Method 3: Quasistatic adiabatic compression from 100 kPa to 250 kPa in one compressor followed by constant pressure cooling to 300 K at 250 kPa, and then quasistatic adiabatic compression from 250 kPa to 800 kPa in a second compressor.

Determine the method in which the total power requirement is the lowest.

Solution to Example 10.11

Method 1:

Applying (10.11) to the first compressor operated adiabatically, assuming that air behaves as an ideal gas, we get

$$(\dot{W}_s)_{in} = \dot{m}\, C_p\, (T_e - T_i) \tag{10.28}$$

Substituting the numerical data in the problem and the properties of air from Table 5.2 in (10.28), we get

$$(\dot{W}_s)_{in} = \left(0.05\ \frac{\text{kg}}{\text{s}}\right)\left(1.005\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(T_e - 300)\ \text{K}$$

where T_e is unknown. Since the flow is taken to be quasistatic adiabatic, we can use (7.31) to determine T_e as follows:

$$T_e = T_i\ \left(\frac{P_e}{P_i}\right)^{(\gamma-1)/\gamma} = 300\ \text{K}\ \times\ \left(\frac{800}{100}\right)^{(1.4-1)/1.4} = 543.4\ \text{K}$$

Therefore,

$$(\dot{W}_s)_{in} = 12.23\ \text{kJ/s} = 12.23\ \text{kW}$$

That is, the power requirement of a single compressor to adiabatically and quasistatically compress air at 100 kPa and 300 K to 800 kPa is 12.23 kW.

Method 2:

Here, air is compressed to 250 kPa in one compressor, and the hot air leaving it is compressed in a second compressor to 800 kPa. Both compressions are quasistatic adiabatic. The power requirement of the first compressor can be worked out in a manner similar to that is described under Method 1, except for the fact that the exit pressure is now 250 kPa, not 800 kPa. Therefore, we get the exit temperature as

$$T_e = T_i \left(\frac{P_e}{P_i}\right)^{(\gamma-1)/\gamma} = 300\text{ K} \times \left(\frac{250}{100}\right)^{(1.4-1)/1.4} = 389.8\text{ K}$$

Substituting the numerical value of T_e in (10.28), we get $(\dot{W}_s)_{in} = 4.51$ kW.

The second compressor works in a way very similar to the first, except for that the inlet pressure is 250 kPa, the inlet temperature is 389.8 K, and the exit pressure is 800 kPa. Therefore, we get the exit temperature of the second compressor as

$$T_e = 389.8\text{ K} \times \left(\frac{800}{250}\right)^{(1.4-1)/1.4} = 543.5\text{ K}$$

The power requirement of the second compressor, evaluated using (10.28), is $(\dot{W}_s)_{in} = 7.72$ kJ/s $= 7.72$ kW.

The total power requirement of the two compressors is obtained by adding 4.51 kW to 7.72 kW, which is 12.23 kW.

Method 3:

The first compressor here is similar to the first compressor in Method 2, and it's power requirement is 4.51 kW. The second compressor here has an inlet temperature of 300 K, and

$$T_e = 300\text{ K} \times \left(\frac{800}{250}\right)^{(1.4-1)/1.4} = 418.3\text{ K}$$

and the power consumption, evaluated using (10.28), is 5.94 kW.

The total power requirement of the two compressors is obtained by adding 4.51 kW to 5.94 kW, which is therefore 10.45 kW.

The total power consumptions are the same for Method 1 and for Method 2. The total power consumption is the lowest for Method 3, in which the hot air exiting the first compressor is cooled to 300 K before it is fed to the second compressor. Method 3 uses the practice known as **multi-stage compression with intercooling** in order to decrease the work required to compress the gas between two specified pressures.

Example 10.12

An ideal gas is compressed by an adiabatic compressor in a steady-flow process, and cooled to its initial temperature. The potential and kinetic energy changes are negligible. Compare the heat removed from the gas in the cooler with the work done on the gas by the compressor.

Solution to Example 10.12

The schematic of the given system is shown in Figure 10.11.

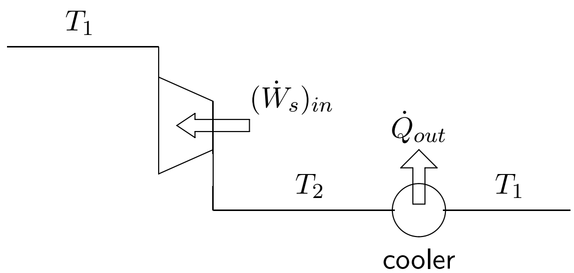


Figure 10.11 Schematic for Example 10.12.

Equation (10.11) applied to the ideal gas flow through the adiabatic compressor becomes

$$(\dot{W}_s)_{in} = \dot{m}\, C_p\, (T_2 - T_1) \tag{10.29}$$

where T_1 and T_2 are the respective inlet and exit temperatures of the gas flowing through the compressor, as shown in Figure 10.11.

The term $(\dot{W}_s)_{in}$ is a positive quantity since work is always done on the gas by the compressor. Consequently, the gas temperature T_2 at the compressor exit is always larger than the temperature T_1 at the inlet. The gas leaving the compressor at T_2 is cooled to its initial temperature T_1 in a cooler. The steady flow energy equation (10.3), when applied to the cooler becomes

$$\dot{Q}_{in} = \dot{m}\,(h_e - h_i)$$

since there is no work exchange and since the potential and kinetic energy changes are neglected.

For an ideal gas, the above equation becomes $\dot{Q}_{in} = \dot{m}\, C_p\, (T_1 - T_2)$. Since the cooler exit temperature T_1 is less than the inlet temperature T_2, the above equation can be rewritten as

$$\dot{Q}_{out} = \dot{m}\, C_p\, (T_2 - T_1) \tag{10.30}$$

which is a positive quantity.

Combining (10.29) and (10.30), we get $(\dot{W}_s)_{in} = \dot{Q}_{out}$. That is, the work done on the gas by the compressor equals the heat removed from the gas in the cooler under the conditions stated in the problem.

Example 10.13

A pump is used to increase the pressure of 25 kg/s of saturated water at 0.07 bar entering the pump to 100 bar and 40°C. Determine the power input to the pump assuming adiabatic flow through the pump.

Solution to Example 10.13

Neglecting the potential and kinetic energy changes, the power input to the adiabatic pump can be determined using (10.11), where h_i is the specific enthalpy of saturated water at 0.07 bar and h_e is the specific enthalpy at 50 bar and 40°C. From a Saturated Water and Steam Table, we find $h_i = 163$ kJ/kg. The water leaving the pump is at compressed state at 100 bar. From a Compressed Water Table*, we find $h_e = 176$ kJ/kg. Substituting the numerical values known in (10.11), the power input of the pump can be calculated as $\dot{W}_{in} = 25\, \times\, (176 - 163)$ kJ/s $= 325$ kW.

Example 10.14

Wet steam at 7 bar is throttled adiabatically to 1 bar and 110°C. Determine the dryness fraction of the wet steam at 7 bar.

*Table A-7 of Çengel, Y.A. & Boles, M.A. 1998 Thermodynamics: an engineering approach, 3^{rd} Edition, McGraw-Hill International Editions.

Solution to Example 10.14

Steam flow through the adiabatic throttling valve can be expressed by (10.13) which states that the specific enthalpies at the inlet h_i and at the exit h_e are nearly the same. The value of h_e at 1 bar and 110°C can be obtained from a Superheated Steam Table as 2696 kJ/kg. According to (10.13), the value of h_i at 7 bar is the same as 2696 kJ/kg. At 7 bar, a Saturated Water and Steam Table gives $h_f = 697$ kJ/kg and $h_g = 2764$ kJ/kg. Therefore, the dryness fraction of the wet steam at 7 bar can be calculated as

$$x = \frac{h_e - h_f}{h_g - h_f} = \frac{2696 - 697}{2764 - 697} = 0.967 = 96.7\%$$

Example 10.15

Water flowing at 5 bar and 120°C is mixed with superheated steam flowing at 10 bar and 200°C in an adiabatic mixing chamber to produce saturated water at 8 bar. Determine the ratio of the mass flow rates of water and the superheated steam, neglecting the changes in kinetic and potential energies.

Solution to Example 10.15

The steady flow energy equation applied to the adiabatic mixing chamber shown in Figure 10.7, neglecting the changes in potential and kinetic energies, is given by (10.16), where $i1$ denotes the water entering the mixing chamber at 5 bar and 120°C, $i2$ denotes the superheated steam entering the mixing chamber at 10 bar and 200°C, and e denotes the saturated steam leaving the mixing chamber at 8 bar.

The water at 5 bar and 120°C is at compressed state. Since the pressure is 5 bar, we use the approximate method discussed in the **Solution to Example (6.8)** to determine h_{i1}. In this method, we approximate h_{i1} to the saturated water specific enthalpy at 120°C, which is 504 kJ/kg as read from a Saturated Water and Steam Table.

The superheated steam specific enthalpy h_{i2} is directly read from a Superheated Steam Table as 2829 kJ/kg at 10 bar and 200°C. The saturated water specific enthalpy h_e is directly read from a Saturated Water and Steam Table as 721 kJ/kg at 8 bar.

Substituting the known numerical values in (10.16), we can determine the ratio of the mass flow rate of water to the mass flow rate of superheated steam as

$$\frac{m_{i1}}{m_{i2}} = \frac{h_{i2} - h_e}{h_e - h_{i1}} = \frac{2829 - 721}{721 - 504} = 9.7$$

Example 10.16

A gas ($\gamma = 1.3$) fed to a turbine at 10 bar and 370°C, is assumed to expand quasistatically and adiabatically to 4 bar as it flows through the turbine. The gas stream exiting the turbine is mixed with a second stream of the same gas flowing at 4 bar and 38°C, in an adiabatic mixing chamber. The mass flow rate of the gas through the turbine is 4 times the mass flow rate of the second gas stream. Determine the temperature of the gas leaving the mixing chamber at 4 bar, neglecting the changes in kinetic and potential energies and assuming that the gas concerned behaves as an ideal gas.

Solution to Example 10.16

The schematic of the given system is shown in Figure 10.12. The temperature of the gas leaving the mixing chamber is to be found, which is the temperature of the gas stream labeled 4 in Figure 10.12.

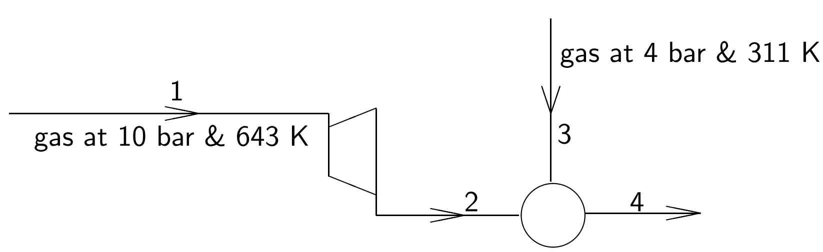


Figure 10.12 Schematic of the system given in Example 10.16.

The steady flow energy equation applied to the gas flow through an adiabatic mixing chamber with negligible changes in the potential and kinetic energies, given by (10.16), becomes

$$\dot{m}_2\,(h_4 - h_2) = \dot{m}_3\,(h_3 - h_4)$$

which is written in accordance with the stream labels shown on Figure 10.12.

Since the gas is assumed to behave as an ideal gas, the above equation becomes

$$\dot{m}_2\,C_p\,(T_4 - T_2) = \dot{m}_3\,C_p\,(T_3 - T_4)$$

which yields

$$T_4 = \frac{\dot{m}_2\,T_2 + \dot{m}_3\,T_3}{\dot{m}_2\, + \dot{m}_3} \tag{10.31}$$

To find T_4, we need the value of each and every term on the right-hand side of (10.31).

First of all note that $\dot{m}_2$ is the mass flow rate through the turbine, and that $\dot{m}_3$ is the mass flow rate of the second gas stream. It is given that the mass flow rate of the gas stream through the turbine is 4 times the mass flow rate of the second gas stream. Therefore, $\dot{m}_2 = 4\,\dot{m}_3$, which reduces (10.31) to

$$T_4 = \frac{4\,T_2 + T_3}{5} \;=\frac{4\,T_2 + 311\text{ K}}{5} \tag{10.32}$$

since $T_3 = 311$ K.

We need to find T_2, which is the temperature of the exit gas stream from the turbine. The flow through the turbine is assumed to be quasistatic adiabatic, and therefore (7.31) can be used to find T_2 as follows:

$$T_2 = T_1\,\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 643\text{ K} \times \left(\frac{4}{10}\right)^{(1.3-1)/1.3} = 520.5\text{ K} = 247.5°\text{C}$$

Substituting $T_2 = 520.5$ K in (10.32), we get $T_4 = 478.6$ K $= 205.6$°C.

Note that the second gas stream is heated from 38°C to 205.6°C by mixing it with the turbine exit at 247.5°C.

Example 10.17

Air ($C_p = 1.005$ kJ/kg · K and $\gamma = 1.4$) is heated as it flows steadily through a pipe of uniform cross-sectional area 150 cm^2. It enters at 300 kPa and 345 K with a velocity of 25 m/s and leaves at 200 kPa and 800 K, and may be assumed to behave as an ideal gas. Determine the amount of heat added per kilogram of air.

If this amount of heat is supplied to 1 kg of air in a closed rigid container at 300 kPa and 345 K, what will be the final temperature and pressure of air in the container?

Solution to Example 10.17

The steady flow energy equation (10.3) applied to the pipe flow yields

$$\dot{Q}_{in} = \dot{m}\left(h_e - h_i + \frac{c_e^2 - c_i^2}{2}\right)$$

since there is no work exchange or potential energy change.

Substituting the given numerical values in the above equation with the assumption that air behaves as an ideal gas, we get

$$\frac{\dot{Q}_{in}}{\dot{m}} = \left[1005\ \frac{\mathsf{J}}{\mathsf{kg \cdot K}} \times (800 - 345)\ \mathsf{K} + \frac{c_e^2 - 25^2}{2}\ \left(\frac{\mathsf{m}}{\mathsf{s}}\right)^2\right] \tag{10.33}$$

where the exit velocity c_e is unknown.

To determine the exit velocity, let us use the fact that the flow is steady, and therefore the mass flow rate is the same at the inlet and the exit. Thus (10.24) gives

$$c_e = \left(\frac{A_i}{A_e}\right)\left(\frac{P_i}{P_e}\right)\left(\frac{T_e}{T_i}\right)c_i = \left(\frac{150}{150}\right)\left(\frac{300}{200}\right)\left(\frac{800}{345}\right)25\ \mathsf{m/s} = 87\ \mathsf{m/s}$$

Substituting $c_e = 87$ m/s in (10.33), we get $\dot{Q}_{in} = 460.75$ kJ per kg of air flowing through the pipe.

The second part of the problem states that 460.75 kJ/kg of heat is supplied to air in a closed rigid container at 300 kPa and 345 K. Suppose the mass of air in the close container is m, then $Q_{in} = 460.75\ m$ kJ. Therefore, the first law applied to the given closed system yields,

$$460.75\, m = \Delta U$$

since no work is supplied to the air in the closed rigid container. Since air is taken as an ideal gas, we get

$$\Delta U = m\, C_v\, (T_f - T_i)$$

Therefore,

$$460.75\, m = m\, 0.718\, (T_f - 345)$$

which gives $T_f = 986.7$ K.

The pressure in the closed container can be obtained using the ideal gas equation of state for the given closed system as follows:

$$m = \frac{P_f\, V_f}{R\, T_f} = \frac{P_i\, V_i}{R\, T_i}$$

Since $V_f = V_i$ for the closed rigid container, we get

$$P_f = P_i \left(\frac{T_f}{T_i} \right) = 300 \text{ kPa} \left(\frac{986.7}{345} \right) = 858 \text{ kPa}$$

Note that when the same amount of heat is provided, the temperature and pressure increases in the closed system are far greater than those in the open system. You may try to figure out why it is so as an exercise.

Example 10.18

Compressed air is preheated in a shell-and-tube heat exchanger, shown in Figure 10.13, before it enters the combustion chamber.

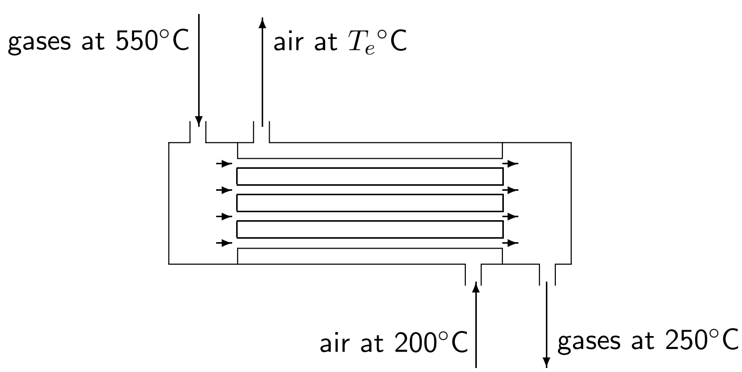


Figure 10.13 Schematic of a heat exchanger of Example 10.18.

It enters the heat exchanger at 9 bar and 200°C with a mass flow rate of 16 kg/s. It gains heat from the exhaust gases leaving a turbine. Exhaust gases enter the heat exchanger at 1.4 bar and 550°C and leave at 1.2 bar and 250°C at a mass flow rate of 17 kg/s. Assume that C_p for the exhaust gases are the same as that for air and that the heat exchanger operates under adiabatic conditions. Determine the exit temperature of the air and the amount of heat transferred from the exhaust gases to the air.

Solution to Example 10.18

For an adiabatic heat exchanger with two fluid streams, (10.19) can be used. Taking the exhaust gases as A and the compressed air as B and assuming ideal gas behaviour, (10.19) can be rewritten as

$$\frac{\dot{m}_{exhaust\ gases}}{\dot{m}_{air}} = \frac{(h_e - h_i)_{air}}{(h_i - h_e)_{exhaust\ gases}} = \frac{(T_e - T_i)_{air}}{(T_i - T_e)_{exhaust\ gases}}$$

since C_p for the exhaust gases is assumed to be the same as that for the air.

Substituting the numerical values known in the above equation, we get

$$\frac{17}{16} = \frac{T_e - 200}{550 - 250} \tag{10.34}$$

which gives the exit temperature of air T_e as 519°C.

To determine the heat transferred from the exhaust gases to the air, let us apply the steady flow energy equation (10.3) to one of the streams. For air,

$$\dot{Q}_{in} = \dot{m}\, C_p\, (T_e - T_i) = 16 \times 1.005 \times (519 - 200) \text{ kJ/s} = 5.13 \text{ MJ/s}$$

Example 10.19

Rework **Example 10.18** with a mass flow rate of air taken as of 14 kg/s, and comment on your results.

Solution to Example 10.19

Replacing 16 in (10.34) by 14, we get

$$\frac{17}{14} = \frac{T_e - 200}{550 - 250} \tag{10.35}$$

which gives the exit air temperature T_e as 564°C.

The exit air temperature of 564°C is greater than the temperature of the exhaust gases entering the heat exchanger, which is 550°C. Since the air leaving the heat exchanger is receiving heat from the exhaust gases entering the heat exchanger, the exhaust gas temperature must always be greater than the air temperature. Therefore, the answer we got is unrealistic. Thus, we conclude that the mass flow rate of air should not be as low as 14 kg/s.

10.8 Summary

- A steady flow process with a single-stream flowing through the system is described by

$$\dot{m}_i = \dot{m}_e \tag{10.1}$$

and by

$$\dot{Q}_{in} + (\dot{W}_s)_{in} = \dot{m}\left[h_e - h_i + \frac{c_e^2 - c_i^2}{2} + g\,(z_e - z_i]\right] \tag{10.3}$$

which is the steady flow energy equation (abbreviated to SFEE) for a single-stream process.

- For a multiple-stream steady flow process, that is, a system with many inlets and many exits for mass to flow in and out of the system, the steady flow energy equation (SFEE) becomes

$$\begin{aligned}\dot{Q}_{in} + (\dot{W}_s)_{in} = & \left[\dot{m}_{e1}\left(h_{e1} + \frac{c_{e1}^2}{2} + gz_{e1}\right)\right] \\ & + \left[\dot{m}_{e2}\left(h_{e2} + \frac{c_{e2}^2}{2} + gz_{e2}\right)\right] + \cdots \\ & - \left[\dot{m}_{i1}\left(h_{i1} + \frac{c_{i1}^2}{2} + gz_{i1}\right)\right] \\ & - \left[\dot{m}_{i2}\left(h_{i2} + \frac{c_{i2}^2}{2} + gz_{i2}\right)\right] - \cdots\end{aligned} \tag{10.4}$$

which is solved together with the mass balance for a multiple-stream steady flow process,

$$\dot{m}_{e1} + \dot{m}_{e1} + \cdots = \dot{m}_{i1} + \dot{m}_{i2} + \cdots \tag{10.5}$$

where the subscripts $e1$, $e2$, $\cdots$ denote exit 1, exit 2, and so on, respectively, and the subscripts $i1$, $i2$, $\cdots$ denote inlet 1, inlet 2, and so on, respectively.

- We can add a quantity in J/kg to a quantity in $(\mathsf{m/s})^2$ since they are equivalent as shown below:

$$\frac{\mathsf{J}}{\mathsf{kg}} = \frac{\mathsf{N \cdot m}}{\mathsf{kg}} = \frac{\mathsf{kg \cdot m}}{\mathsf{s}^2}\,\frac{\mathsf{m}}{\mathsf{kg}} = \left(\frac{\mathsf{m}}{\mathsf{s}}\right)^2$$

- Assume that a fluid flows perpendicular to a cross section with a cross-sectional area A, at a uniform speed c and at a uniform density ρ. The mass flow rate of the fluid through the cross-sectional area A is then

$$\dot{m} = A\,\rho\,c = \frac{Ac}{v} \tag{10.23}$$

where v is the specific volume.

If the fluid were steam or a mixture of water and steam, v could be found from the Steam Table.

If the fluid were assumed to behave as an ideal gas, then the ideal gas equation of state would give $v = RT/P$. Therefore, the mass flow rate of an ideal gas through the cross-sectional area A is

$$\dot{m} = \frac{A\,c\,P}{RT} \tag{10.24}$$